Use a Riemann sum with $m = n = 2$ to estimate the value of

$$\int_R \sin(x + y) \, dA$$

where $R = [0, \pi] \times [0, \pi]$. Take the sample points to be lower left corners. Include a drawing of the rectangle $R$ with the indicated subdivisions and sample points. Also include all of your calculations. (The answer you get should be $\pi^2/2$.)

**Solution:** Note that $\Delta x = \Delta y = \pi/2$. Thus $\Delta A = \Delta x \Delta y = \pi^2/4$. The rectangle, $R$, with sample points indicated is shown below.
The corresponding Riemann sum estimate of the indicated integral is

\[
\begin{align*}
  f(0, 0) \Delta A + f \left( \frac{\pi}{2}, 0 \right) \Delta A + f \left( 0, \frac{\pi}{2} \right) \Delta A + f \left( \frac{\pi}{2}, \frac{\pi}{2} \right) \Delta A \\
  = \left( \sin \left( 0 + 0 \right) + \sin \left( \frac{\pi}{2} + 0 \right) + \sin \left( \frac{\pi}{2} + \frac{\pi}{2} \right) \right) \cdot \frac{\pi^2}{4} \\
  = (0 + 1 + 1 + 0) \cdot \frac{\pi^2}{4} \\
  = \frac{\pi^2}{2}.
\end{align*}
\]