Instructions. Your work on this exam will be graded according to two criteria: mathematical correctness and clarity of presentation. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using complete sentences where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator but you may not use any books or notes.

1. Find an equation for the sphere with center \((-5, -3, 8)\) and radius 10. (This is a very short problem. You can just write down the answer.)

Answer:

\[(x + 5)^2 + (y + 3)^2 + (z - 8)^2 = 100.\]

2. A clothesline is tied between two poles that are 8 m apart. The line is quite taught and has negligible sag. When a wet shirt with mass of 0.8 kg is hung at the middle of the line, the midpoint is pulled down 8 cm. Find the magnitude of the tension in each half of the clothesline.

Answer: See solutions to Section 9.2 Homework problems on course Web page.

3. Find the vector of length 3 that points in the opposite direction of the vector \(v = i + 2k\).

Solution: Since \(|v| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{5}\), then a unit vector that points in the same direction as \(v\) is

\[u = \frac{1}{\sqrt{5}}v = \frac{\sqrt{5}}{5}v.\]

Thus a vector of length 3 that points in the opposite direction of \(v\) is

\[-3u = \frac{-3\sqrt{5}}{5}v.\]

4. Let \(a = 5i - 2j - k\) and \(b = 3k\). Compute \(a \times b\) and show that \(a \times b\) is orthogonal to both \(a\) and \(b\).

Solution:

\[a \times b = (5i - 2j - k) \times 3k = -15j - 6i = -6i - 15j.\]

Also

\[(a \times b) \cdot a = (-6)(5) + (-15)(-2) + (0)(-1) = 0\]

and

\[(a \times b) \cdot b = (-6)(0) + (-15)(0) + (0)(3) = 0.\]
5. Solve the following problem.

A bicycle pedal is pushed by a foot with a 60-N force as shown. The shaft of the pedal is 18 cm long. Find the magnitude of the torque about P.

Solution: The magnitude of the torque is

$$|\tau| = (0.18 \text{ m}) (60 \text{ N}) \sin (80^\circ) \approx 10.64 \text{ Joules.}$$

6. Find the point at which the line

$$x = 5$$
$$y = -5 - 2t$$
$$z = 1 - 4t$$

intersects the plane

$$-3x + 3y = 5.$$

Solution: At the point of intersection, we have

$$-3(5) + 3(-5 - 2t) = 5$$

or

$$-15 - 15 - 6t = 5$$

or

$$-6t = 35$$

or

$$t = -\frac{35}{6}.$$

The point of intersection is thus \((x, y, z)\) where

$$x = 5$$

$$y = -5 - 2 \left( -\frac{35}{6} \right) = \frac{20}{3}$$

$$z = 1 - 4 \left( -\frac{35}{6} \right) = \frac{73}{3}.$$. 
7. Determine whether the quadric surface with equation
\[ x^2 + y^2 - z^2 - x + 2z = \frac{7}{4} \]
is an elliptic paraboloid, a hyperbolic paraboloid, a double cone, an ellipsoid, an hyperboloid of one sheet, or an hyperboloid of two sheets. (You must include all algebraic details in showing how you go about making this determination.)

**Solution:** By completing the square, we obtain
\[ x^2 - x + 1 + y^2 - (z^2 - 2z + 1) = \frac{7}{4} + \frac{1}{4} - 1 \]
or
\[ \left(x - \frac{1}{2}\right)^2 + y^2 - (z - 1)^2 = 1 \]
This shows that the surface is an hyperboloid of one sheet.

8. Find both rectangular and spherical coordinates for the point whose cylindrical coordinates are \((1, \pi, -2)\). Show all details of your work.

**Solution:** First we find rectangular coordinates:

\[
\begin{align*}
x &= 1 \cdot \cos (\pi) = -1 \\
y &= 1 \cdot \sin (\pi) = 0 \\
z &= -2
\end{align*}
\]
so the rectangular coordinates are \((-1, 0, -2)\).

Now we find spherical coordinates:

\[
\rho = \sqrt{(-1)^2 + 0^2 + (-2)^2} = \sqrt{5}
\]
and \(\theta = \pi\), and

\[
\cos (\phi) = \frac{z}{\rho} = \frac{-2}{\sqrt{5}}
\]
so

\[
\phi = \arccos \left(\frac{-2}{\sqrt{5}}\right) \approx 2.68.
\]
The spherical coordinates are thus \((\sqrt{5}, \pi, 2.68)\).