A wrench that is 0.4 meters long lies along the positive $x$ axis and grips a bolt at the origin. A force is applied in the direction of the vector $(-1, -1)$. Find the magnitude of the force needed to supply 100 Joules of torque to the bolt.

**Solution 1 (the hard way):** Recall that the torque is $\tau = \mathbf{r} \times \mathbf{F}$ where $\mathbf{r}$ is the “wrench” vector and $\mathbf{F}$ is the force vector. In this problem, the wrench vector is

$$\mathbf{r} = 0.4\mathbf{i}$$

and the force vector is

$$\mathbf{F} = |\mathbf{F}| \cos (225^\circ) \mathbf{i} + |\mathbf{F}| \sin (225^\circ) \mathbf{j} = -\frac{\sqrt{2}}{2} |\mathbf{F}| (\mathbf{i} + \mathbf{j}).$$

We now see that

$$\tau = \mathbf{r} \times \mathbf{F}$$

$$= (0.4\mathbf{i}) \times \left( \left( -\frac{\sqrt{2}}{2} |\mathbf{F}| \right) (\mathbf{i} + \mathbf{j}) \right)$$

$$= (0.4) \left( -\frac{\sqrt{2}}{2} |\mathbf{F}| \right) (\mathbf{i} \times (\mathbf{i} + \mathbf{j}))$$

$$= -0.2\sqrt{2} |\mathbf{F}| (\mathbf{i} \times \mathbf{i} + \mathbf{i} \times \mathbf{j})$$

$$= -0.2\sqrt{2} |\mathbf{F}| (\mathbf{0} + \mathbf{k})$$

$$= -0.2\sqrt{2} |\mathbf{F}| \mathbf{k}.$$ 

Hence

$$|\tau| = |\mathbf{r} \times \mathbf{F}| = \left| -0.2\sqrt{2} |\mathbf{F}| \mathbf{k} \right| = 0.2\sqrt{2} |\mathbf{F}|.$$ 

In order to have $|\tau| = 100$ Joules, we must have

$$|\mathbf{F}| = \frac{100}{0.2\sqrt{2}} \approx 353.55 \text{ Newtons.}$$
Solution 2 (the easy way): Recall that the magnitude of the torque is $|\tau| = |r| |F| \sin(\theta)$ where $r$ is the “wrench” vector, $F$ is the force vector, and $\theta$ is the angle between $r$ and $F$. In this problem, the wrench vector is $r = 0.4i$ and hence $|r| = 0.4$. Also $\theta = 135^\circ$. Therefore

$$|\tau| = 0.4 |F| \sin(135^\circ)$$

or

$$|\tau| = 0.2\sqrt{2} |F|.$$

In order to have $|\tau| = 100$ Joules, we must have

$$|F| = \frac{100}{0.2\sqrt{2}} \approx 353.55 \text{ Newtons.}$$