Consider the plane curve described by the vector-valued function
\[ \mathbf{r}(t) = -3t\mathbf{i} + (2t - 3)\mathbf{j}. \]

1. Sketch the graph of this curve over the parameter interval \(-2 \leq t \leq 2\).
2. Compute \(\mathbf{r}'(t)\).
3. Compute the arc length, \(s\), of this curve (measured from \(t = 0\)):
\[ s = \int_{0}^{t} |\mathbf{r}'(u)| \, du = ? \]
4. Reparameterize this curve with respect to arc length measured from the point where \(t = 0\) in the direction of increasing \(t\).

**Solution:** The graph of the curve over the parameter interval \(-2 \leq t \leq 2\) is a line segment that begins at the point \((6, -7)\) and ends at the point \((-6, 1)\). Also
\[ \mathbf{r}'(t) = -3\mathbf{i} + 2\mathbf{j} \]
and
\[ |\mathbf{r}'(t)| = \sqrt{(-3)^2 + (2)^2} = \sqrt{13}. \]
Thus the arc length of the curve over the parameter interval \([0, t]\) is
\[ s = \int_{0}^{t} \sqrt{13} \, du = \sqrt{13}t. \]
The reparameterization of the curve with respect to arc length is
\[ \mathbf{p}(s) = -3 \left( \frac{s}{\sqrt{13}} \right) \mathbf{i} + \left( 2 \left( \frac{s}{\sqrt{13}} \right) - 3 \right) \mathbf{j} = -\frac{3\sqrt{13}}{13} s \mathbf{i} + \left( \frac{2\sqrt{13}}{13} s - 3 \right) \mathbf{j}. \]
Please understand that the curves described by $r$ and $p$ are the same! For example the portion of $r$ that is traced out as $t$ varies over the interval $-2 \leq t \leq 2$ is the same as the portion of $p$ that is traced out as $s$ varies over the interval $-2\sqrt{13} \leq t \leq 2\sqrt{13}$. What is “special” about the parameter $s$ is that it corresponds to the actual distance travelled along the curve (beginning from the point $(6, -7)$) in travelling to the point whose position vector is $p(s)$. 