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Let $\mathbf{F}$ be the vector field

$$\mathbf{F}(x, y) = -32 \mathbf{j}$$

and let $C$ be the parabolic curve $y = x^2$ beginning at the point $(0, 0)$ and ending at the point $(1, 1)$.

Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

You must include all details of your work.

**Solution:** The easiest way to do this is to use the Fundamental Theorem of Line Integrals: The vector field $\mathbf{F}(x, y) = -32 \mathbf{j}$ is conservative and has potential function $f(x, y) = -32y$. Thus

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f(1, 1) - f(0, 0) = -32 - 0 = -32.$$

Now we show how to do the integral using the definition of line integrals: The curve $C$ can be parameterized as

$$\mathbf{r}(t) = ti + t^2 \mathbf{j}$$

$$0 \leq t \leq 1$$

from which we see that

$$\mathbf{r}'(t) = \mathbf{i} + 2t \mathbf{j}$$

and

$$\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = (-32 \mathbf{j}) \cdot (\mathbf{i} + 2t \mathbf{j}) = -64t.$$

Thus

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt = \int_0^1 -64t \, dt = -32.$$