Let $\mathbf{F}$ be the vector field

$$\mathbf{F}(x, y) = \mathbf{i} + \mathbf{j}$$

and let $C$ be the directed line segment beginning at the point $(0, 0)$ and ending at the point $(1, 1)$.

Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$ 

You must include all details of your work.

**Solution:** The easiest way to do this is to use the Fundamental Theorem of Line Integrals: The vector field $\mathbf{F}(x, y) = \mathbf{i} + \mathbf{j}$ is conservative and has potential function $f(x, y) = x + y$. Thus

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f(1, 1) - f(0, 0) = 2 - 0 = 2.$$ 

Now we show how to do the integral using the definition of line integrals: The curve $C$ can be parameterized as

$$\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j}$$

from which we see that

$$\mathbf{r}'(t) = \mathbf{i} + \mathbf{j}$$

and

$$\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = (\mathbf{i} + \mathbf{j}) \cdot (\mathbf{i} + \mathbf{j}) = 2.$$ 

Thus

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt = \int_0^1 2 \, dt = 2.$$