1. (a) Use a Riemann sum with \( m = n = 2 \) to estimate the value of

\[
\iint_R \sin(x + y) \, dA
\]

where \( R = [0, \pi] \times [0, \pi] \). Take the sample points to be the lower left corners.

**Solution:** We have

\[
\Delta x = \frac{\pi - 0}{2} = \frac{\pi}{2}
\]

and likewise \( \Delta y = \pi/2 \). Therefore

\[
\Delta A = \Delta x \Delta y = \frac{\pi^2}{4}.
\]

The lower left corners of the subrectangles are \((0,0), (\pi/2,0), (0,\pi/2), \) and \((\pi/2, \pi/2)\). Therefore the Riemann sum estimate is

\[
\frac{\pi^2}{4} \left( \sin(0 + 0) + \sin \left( \frac{\pi}{2} + 0 \right) + \sin \left( 0 + \frac{\pi}{2} \right) + \sin \left( \frac{\pi}{2} + \frac{\pi}{2} \right) \right) = \frac{\pi^2}{4} (2) = \frac{\pi^2}{2}.
\]

(b) Obtain another estimate for this integral by using the midpoints as sample points.

**Solution:** The midpoints of the subrectangles are \((\pi/4, \pi/4), (3\pi/4, \pi/4), (\pi/4, 3\pi/4), \) and \((3\pi/4, 3\pi/4)\). Therefore the Riemann sum estimate is

\[
\frac{\pi^2}{4} \left( \sin \left( \frac{\pi}{4} + \frac{\pi}{4} \right) + \sin \left( \frac{3\pi}{4} + \frac{\pi}{4} \right) + \sin \left( \frac{\pi}{4} + \frac{3\pi}{4} \right) + \sin \left( \frac{3\pi}{4} + \frac{3\pi}{4} \right) \right) = 0.
\]
2. Use an iterated integral to find the exact value of the integral given in part 1.

**Solution:** The exact value of the given integral is

\[
\int_0^\pi \int_0^\pi \sin (x + y) \ dy \ dx.
\]

Evaluating the inner integral, we obtain

\[
\int_0^\pi \sin (x + y) \ dy = -\cos (x + y) \bigg|_{y=0}^{y=\pi}
\]

\[
= -\cos (x + \pi) + \cos (x)
\]

\[
= - (\cos (x) \cos (\pi) - \sin (x) \sin (\pi)) + \cos (x)
\]

\[
= 2 \cos (x).
\]

This gives

\[
\int_0^\pi \int_0^\pi \sin (x + y) \ dy \ dx = \int_0^\pi 2 \cos (x) \ dx
\]

\[
= 2 \sin (x) \bigg|_{x=0}^{x=\pi}
\]

\[
= 0.
\]

Here is a graph of \( f (x, y) = \sin (x + y) \). Note that the equal parts of the graph lie above and below the plane \( z = 0 \). This explains why the integral is equal to 0.