Instructions. Your work on this exam will be graded according to two criteria: **mathematical correctness** and **clarity of presentation**. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using **complete sentences** where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator but you may not use any books or notes.

1. Match the curves defined by \( r(t) \) in a–e with the pictures of these curves given in 1–5.

(a) \( r(t) = (\sin(t), t), -\pi \leq t \leq \pi \).

(b) \( r(t) = t^2i + t^4j + t^6k, 0 \leq t \leq 1 \).

(c) \( r(t) = (1 + t, 3t, -t), -1 \leq t \leq 1 \)

(d) \( r(t) = (1, \cos(t), 2 \sin(t)), 0 \leq t \leq 2\pi \)

(e) \( r(t) = (t^3, t^2), -1 \leq t \leq 1 \)

2. Find parametric equations for the tangent line to the curve

\[
\begin{align*}
x &= t^5 \\
y &= t^4 \\
z &= t^3
\end{align*}
\]

at the point \((1, 1, 1)\). (Be detailed in your explaining your procedure.)

**Solution:** We can write the equation of this curve in vector form as

\[ r(t) = t^5i + t^4j + t^3k. \]

The tangent vector to this curve (at a general point) is

\[ r'(t) = 5t^4i + 4t^3j + 3t^2k. \]

Therefore the tangent vector at the point \((1, 1, 1)\) (which corresponds to \( t = 1 \)) is

\[ r'(1) = 5i + 4j + 3k. \]

This tangent vector is parallel to the tangent line (and is thus a “direction vector” for the tangent line). Thus, parametric equations for the tangent line are

\[
\begin{align*}
x &= 1 + 5t \\
y &= 1 + 4t \\
z &= 1 + 3t.
\end{align*}
\]

The curve and the tangent line that we just found are pictured below.
3. A graph of the function
\[ y = 4x^{5/2} \]
is shown below. Find the curvature function, \( \kappa \), for \( f \). There are a couple of different ways to go about this, but one way seems to be easier than the others. Be detailed.

**Solution:** The curvature function is
\[ \kappa(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}. \]
Since 
\[ f'(x) = 10x^{3/2} \]
and 
\[ f''(x) = 15x^{1/2}, \]
we obtain 
\[ (f'(x))^2 = 100x^3 \]
and thus 
\[ \kappa(x) = \frac{15\sqrt{x}}{(1 + 100x^3)^{3/2}}. \]

The curvature function is graphed below.

4. Find the velocity, acceleration, and speed functions of a particle whose path of motion is given by
\[ \mathbf{r}(t) = t\mathbf{i} + 2\cos(t)\mathbf{j} + \sin(t)\mathbf{k}. \]

**Solution:** The velocity (vector) function of this particle is
\[ \mathbf{v}(t) = \mathbf{r'}(t) = \mathbf{i} - 2\sin(t)\mathbf{j} + \cos(t)\mathbf{k} \]
and the acceleration (vector) function is
\[ \mathbf{a}(t) = \mathbf{v'}(t) = -2\cos(t)\mathbf{j} - \sin(t)\mathbf{k}. \]

The (scalar) speed function is
\[ v(t) = |\mathbf{v}(t)| = \sqrt{1 + 4\sin^2(t) + \cos^2(t)}. \]

5. Match the parametrically-defined surfaces in a–e with the pictures of these surfaces given in I–V.

(a) \[ \mathbf{r}(u,v) = (u + v)\mathbf{i} + (3 - v)\mathbf{j} + (1 + 4u + 5v)\mathbf{k}, \quad -1 \leq u \leq 1, \quad -1 \leq v \leq 1. \]
(b) \( \mathbf{r} (\theta, z) = 2 \sin (\theta) \mathbf{i} + 3 \cos (\theta) \mathbf{j} + z \mathbf{k}, \ 0 \leq \theta \leq 2\pi, \ 0 \leq z \leq 2. \)

(c) \( \mathbf{r} (s, t) = (s, t, t^2 - s^2), \ -1 \leq s \leq 1, \ -1 \leq t \leq 1. \)

(d) \( \mathbf{r} (t, y) = (y \sin (2t), y^2, y \cos (2t)), \ 0 \leq t \leq \pi, \ 0 \leq y \leq 2. \)

(e) \( \mathbf{r} (\theta, \alpha) = (4 \cos (\theta) + \cos (\alpha) \cos (\theta), 4 \sin (\theta) + \cos (\alpha) \sin (\theta), \sin (\alpha)), \ 0 \leq \theta \leq 2\pi, \ 0 \leq \alpha \leq 2\pi. \)

Graphs for Problem 1 (Graphs of Curves)
Graph 2 (matches b)

Graph 3 (matches d)
Graphs for Problem 5 (Graphs of Surfaces)
Graph 1 (matches a)

Graph 2 (matches e)
Graph III (matches b)

Graph IV (matches c)
Graph V (matches d)