Find an equation for the plane that contains the point \( P_0 (-9, 0, 2) \) and also contains the line, \( L \), with parametric equations

\[
\begin{align*}
x &= 2 - 3t \\
y &= 5 \\
z &= 2 - t.
\end{align*}
\]

After you have found an equation for this plane, show how you check to see that your answer is correct.

**Solution:** By setting \( t = 0 \) and \( t = 1 \), we see that the points \( P_1 (2, 5, 2) \) and \( P_2 (-1, 5, 1) \) are on the line \( L \). Since \( \overrightarrow{P_0P_1} = \langle 11, 5, 0 \rangle \) and \( \overrightarrow{P_0P_2} = \langle 8, 5, -1 \rangle \), a vector that is perpendicular to the plane in question is

\[
\mathbf{n} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 11 & 5 & 0 \\ 8 & 5 & -1 \end{vmatrix} = -5\mathbf{i} + 11\mathbf{j} + 15\mathbf{k}.
\]

If \( P(x, y, z) \) is any point in the plane in question, then \( \mathbf{n} \cdot \overrightarrow{P_0P} = 0 \). Thus, an equation for this plane is

\[
\langle -5, 11, 15 \rangle \cdot \langle x + 9, y, z - 2 \rangle = 0
\]

or

\[
-5 (x + 9) + 11y + 15(z - 2) = 0
\]

or

\[
-5x - 45 + 11y + 15z - 30 = 0
\]

or

\[
-5x + 11y + 15z = 75.
\]

Let us check to see that our answer is correct:

\[
-5 (-9) + 11 (0) + 15 (2) = 75
\]

shows that the point \( P_0 \) is in this plane.

\[
-5 (2 - 3t) + 11 (5) + 15 (2 - t) = -10 + 15t + 55 + 30 - 15t = 75
\]

shows that the line \( L \) is contained in this plane.