Instructions. Your work on this exam will be graded according to two criteria: mathematical correctness and clarity of presentation. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using complete sentences where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator but you may not use any books or notes.

1. Find the limit
\[
\lim_{(x,y)\to(0,0)} \frac{xy + x^2}{x^2 + y^2}
\]
if it exists or show that this limit does not exist. (If the limit does not exist, then you must show that there are two different paths of approach to (0,0) for which different limits are obtained. If the limit does exist, then you must provide some reasoning that explains why the limit exists. Be sure to write in sentences and include details.)

Explanation: If we let \((x, y)\) approach \((0,0)\) along the line \(x = 0\), then we obtain
\[
\lim_{(x,y)\to(0,0)\atop(x=0)} \frac{xy + x^2}{x^2 + y^2} = \lim_{y\to0} \frac{0}{y^2} = 0
\]
and if we let \((x, y)\) approach \((0,0)\) along the curve \(y = 0\), then we obtain
\[
\lim_{(x,y)\to(0,0)\atop(y=0)} \frac{xy + x^2}{x^2 + y^2} = \lim_{x\to0} \frac{x^2}{x^2} = \lim_{x\to0} 1 = 1.
\]
Since two different limits can be obtained using different paths of approach to \((0,0)\), we conclude that the limit in question does not exist.

2. For the function
\[
f(x,y) = y \ln(x),
\]
compute \(\partial f/\partial x\) and \(\partial f/\partial y\).

Solution:
\[
\frac{\partial f}{\partial x} = \frac{y}{x}
\]
and
\[
\frac{\partial f}{\partial y} = \ln(x).
\]
3. Find an equation for the tangent plane to the surface

\[ f(x, y) = \frac{x}{y} \]

at the point on the surface corresponding to \((x, y) = (6, 3)\).

**Solution:**

\[ \frac{\partial f}{\partial x} = \frac{1}{y} \]
\[ \frac{\partial f}{\partial y} = -\frac{x}{y^2} \]

so

\[ \frac{\partial f}{\partial x}(6, 3) = \frac{1}{3} \]
\[ \frac{\partial f}{\partial y}(6, 3) = -\frac{2}{3} \]

Also, \((x, y) = (6, 3)\) corresponds to the point \((x_0, y_0, z_0) = (6, 3, 2)\) on the surface, so an equation of the tangent plane at this point is

\[ z - 2 = \frac{1}{3} (x - 6) - \frac{2}{3} (y - 3) \]

or

\[ 3z - 6 = (x - 6) - 2(y - 3) \]

or

\[ 3z - 6 = x - 6 - 2y + 6 \]

or

\[ x - 2y - 3z = -6. \]

4. For

\[ z = e^r \cos (\theta) \]
\[ r = st \]
\[ \theta = \sqrt{s^2 + t^2} \]

use the Chain Rule to find \(\partial z/\partial s\) and \(\partial z/\partial t\). (No need to simplify your answers.)

**Solution:**

\[ \frac{\partial z}{\partial s} = \frac{\partial z}{\partial r} \frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta} \frac{\partial \theta}{\partial s} \]
\[ = (e^r \cos (\theta)) (t) + (-e^r \sin (\theta)) \left( \frac{s}{\sqrt{s^2 + t^2}} \right) \]

and

\[ \frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} \frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta} \frac{\partial \theta}{\partial t} \]
\[ = (e^r \cos (\theta)) (s) + (-e^r \sin (\theta)) \left( \frac{t}{\sqrt{s^2 + t^2}} \right) \]
5. For the function 
\[ f(x, y) = x^2 + y^2 - 2x - 4, \]

(a) Find \( \nabla f(0, 0) \).
(b) For the unit vector \( u = \left\langle \frac{-1}{2}, \frac{\sqrt{3}}{2} \right\rangle \), find \( D_u f(0, 0) \).

**Solution:**

\[ \nabla f(x, y) = f_x(x, y) \mathbf{i} + f_y(x, y) \mathbf{j} \]
\[ = (2x - 2) \mathbf{i} + 2y \mathbf{j} \]

so

\[ \nabla f(0, 0) = -2\mathbf{i}. \]

Also

\[ D_u f(0, 0) = \nabla f(0, 0) \cdot u = (-2) \left\langle \frac{-1}{2} \right\rangle + (0) \left\langle \frac{\sqrt{3}}{2} \right\rangle = 1. \]