The curve \( r(t) = t \cos(t) \mathbf{i} + t \sin(t) \mathbf{j} \), \(-\pi \leq t \leq \pi\) is pictured below.

Graph of \( r(t) = t \cos(t) \mathbf{i} + t \sin(t) \mathbf{j} \), \(-\pi \leq t \leq \pi\)

Show that the curvature of this curve at the point \((0, 0)\) is 2.

**Solution:** The point \((0, 0)\) corresponds to \(t = 0\). The curvature at this point is given by

\[
\kappa(0) = \frac{|r'(0) \times r''(0)|}{|r'(0)|^3}.
\]

Note that

\[
r'(t) = (-t \sin(t) + \cos(t)) \mathbf{i} + (t \cos(t) + \sin(t)) \mathbf{j}
\]

\[
r''(t) = (-t \cos(t) - 2 \sin(t)) \mathbf{i} + (-t \sin(t) + 2 \cos(t)) \mathbf{j}
\]

so

\[
r'(0) = \mathbf{i}
\]

\[
r''(t) = 2j.
\]
This gives us
\[ r'(0) \times r''(0) = \mathbf{i} \times 2\mathbf{j} = 2\mathbf{k} \]
and hence
\[ |r'(0) \times r''(0)| = 2. \]
Thus
\[ \kappa(0) = \frac{2}{1^3} = 2. \]