NAME________________________________

Let \( \mathbf{F} \) be the vector field
\[
\mathbf{F}(x, y) = e^{x-1} \mathbf{i} + xy \mathbf{j}
\]
and let \( C \) be the directed line segment beginning at the point \((0,0)\) and ending at the point \((1,1)\).

Evaluate the line integral
\[
\int_C \mathbf{F} \cdot d\mathbf{r}.
\]
You must include all details of your work.

**Solution:** First note that the given vector field is not conservative, so we cannot use the Fundamental Theorem of Line Integrals. We will have to use the definition of the line integral of a vector field. The curve \( C \) can be parameterized as
\[
\mathbf{r}(t) = ti + tj
\]
with \(0 \leq t \leq 1\)
from which we see that
\[
\mathbf{r}'(t) = \mathbf{i} + \mathbf{j}
\]
and
\[
\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = (e^{t-1}i + t^2j) \cdot (i + j) = e^{t-1} + t^2.
\]
Thus
\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt
= \int_0^1 (e^{t-1} + t^2) \, dt
= e^{t-1} + \frac{1}{3}t^3\bigg|_{t=0}^{t=1}
= (1 + \frac{1}{3}) - (e^{-1} + 0)
= \frac{4}{3} - e^{-1}.
\]