2. 
\[ r(1) = \langle 1, 1 \rangle \]
\[ r(1.1) = \langle 1.21, 1.1 \rangle \]
\[ r(1.1) - r(1) = \langle 0.21, 0.1 \rangle \]
\[ \frac{r(1.1) - r(1)}{0.1} = \langle 2.1, 1 \rangle \]
\[ r'(1) = \langle 2, 1 \rangle \]

Since \( h = 0.1 \) is quite small (close to zero), we have
\[ \frac{r(1.1) - r(1)}{0.1} \approx r'(1). \]

3. For \( r(t) = \langle \cos(t), \sin(t) \rangle \), we have \( r'(t) = \langle -\sin(t), \cos(t) \rangle \). Thus
\[ r'(\pi/4) = \langle -\sin(\pi/4), \cos(\pi/4) \rangle = \langle -\sqrt{2}/2, \sqrt{2}/2 \rangle. \]

Also,
\[ r(\pi/4) = \langle \cos(\pi/4), \sin(\pi/4) \rangle = \langle \sqrt{2}/2, \sqrt{2}/2 \rangle. \]
5. For \( r(t) = (1 + t)i + t^2j \), we have \( r'(t) = i + 2j \). Thus

\[
r'(1) = i + 2j.
\]

Also,

\[
r(1) = 2i + j.
\]

7. For \( r(t) = e^t i + e^{-2t}j \), we have \( r'(t) = e^t i - 2e^{-2t}j \). Thus

\[
r'(0) = i - 2j.
\]

Also,

\[
r(0) = i + j.
\]
9. For \( \mathbf{r}(t) = \langle t^2, 1 - t, \sqrt{t} \rangle \), we have
\[
\mathbf{r}'(t) = \left\langle 2t, -1, \frac{1}{2\sqrt{t}} \right\rangle.
\]

11. For \( \mathbf{r}(t) = e^{t^2} \mathbf{i} - \mathbf{j} + \ln(1 + 3t) \mathbf{k} \), we have
\[
\mathbf{r}'(t) = 2te^{t^2} \mathbf{i} + \frac{3}{1 + 3t} \mathbf{k}.
\]

13. For \( \mathbf{r}(t) = a + t \mathbf{b} + t^2 \mathbf{c} \), we have
\[
\mathbf{r}'(t) = \mathbf{b} + 2t \mathbf{c}.
\]

15. For \( \mathbf{r}(t) = \cos(t) \mathbf{i} + 3t \mathbf{j} + 2\sin(2t) \mathbf{k} \), we have
\[
\mathbf{r}'(t) = -\sin(t) \mathbf{i} + 3 \mathbf{j} + 4\cos(2t) \mathbf{k}.
\]
Thus
\[
\mathbf{r}'(0) = 3 \mathbf{j} + 4 \mathbf{k}
\]
and
\[
\mathbf{T}(0) = \frac{1}{|\mathbf{r}'(0)|} \mathbf{r}'(0) = \frac{1}{5} (3 \mathbf{j} + 4 \mathbf{k}) = \frac{3}{5} \mathbf{j} + \frac{4}{5} \mathbf{k}.
\]

17. For \( \mathbf{r}(t) = \langle t, t^2, t^3 \rangle \), we have
\[
\mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle
\]
\[
\mathbf{r}'(1) = \langle 1, 2, 3 \rangle
\]
\[
|\mathbf{r}'(1)| = \sqrt{14}
\]
\[
\mathbf{T}(1) = \frac{1}{|\mathbf{r}'(1)|} \mathbf{r}'(1) = \left\langle \frac{\sqrt{14}}{14}, \frac{2\sqrt{14}}{14}, \frac{3\sqrt{14}}{14} \right\rangle
\]
\[
\mathbf{r}''(t) = \langle 0, 2, 6t \rangle.
\]
\[ \mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = 6t^2 \mathbf{i} - 6t \mathbf{j} + 2\mathbf{k}. \]

19. The curve with parametric equations
\[\begin{align*}
x &= t^5 \\
y &= t^4 \\
z &= t^3
\end{align*}\]
has tangent vector \( \langle 5t^4, 4t^3, 3t^2 \rangle \). At the point \((1, 1, 1)\) (which is on the curve), we have \( t = 1 \). Thus, the tangent vector at this point is \( \langle 5, 4, 3 \rangle \). The tangent line at this point is the line with parametric equations
\[\begin{align*}
x &= 1 + 5t \\
y &= 1 + 4t \\
z &= 1 + 3t.
\end{align*}\]

21. The curve with parametric equations
\[\begin{align*}
x &= e^{-t} \cos(t) \\
y &= e^{-t} \sin(t) \\
z &= e^{-t}
\end{align*}\]
has tangent vector 
\[ e^{-t} \langle -\sin(t) - \cos(t), \cos(t) - \sin(t), -1 \rangle. \]
At the point \((1, 0, 1)\) (which is on the curve), we have \( t = 0 \). Thus, the tangent vector at this point is \( \langle -1, 1, -1 \rangle \). The tangent line at this point is the line with parametric equations
\[\begin{align*}
x &= 1 - t \\
y &= t \\
z &= 1 - t.
\end{align*}\]

23. The curve with parametric equations
\[\begin{align*}
x &= t \\
y &= \sqrt{2} \cos(t) \\
z &= \sqrt{2} \sin(t)
\end{align*}\]
has tangent vector
\[ \langle 1, -\sqrt{2} \sin(t), \sqrt{2} \cos(t) \rangle. \]
At the point \((\pi/4, 1, 1)\) (which is on the curve), we have \( t = \pi/4 \). Thus, the tangent vector at this point is \( \langle 1, -1, 1 \rangle \). The tangent line at this point is the line with parametric equations
\[ x = \frac{\pi}{4} + t \]
\[ y = 1 - t \]
\[ z = 1 + t. \]

25.

a. For \( \mathbf{r}(t) = \langle t^3, t^4, t^5 \rangle \), we have
\[ \mathbf{r}'(t) = \langle 3t^2, 4t^3, 5t^4 \rangle. \]
The curve defined by \( \mathbf{r} \) is not smooth because \( \mathbf{r}'(0) = \mathbf{0} \).

b. For \( \mathbf{r}(t) = \langle t^3 + t, t^4, t^5 \rangle \), we have
\[ \mathbf{r}'(t) = \langle 3t^2 + 1, 4t^3, 5t^4 \rangle. \]
The curve defined by \( \mathbf{r} \) is smooth because there is not point on this curve at which \( \mathbf{r}'(0) = \mathbf{0} \).

c. For \( \mathbf{r}(t) = \langle \cos^3(t), \sin^3(t) \rangle \), we have
\[ \mathbf{r}'(t) = \langle -3 \cos^2(t) \sin(t), 3 \sin^2(t) \cos(t) \rangle. \]
The curve defined by \( \mathbf{r} \) is not smooth because there are points on this curve at which \( \mathbf{r}'(0) = \mathbf{0} \). (See the graph below.)
Graph of $x = \cos^3(t), y = \sin^3(t)$

27. The curve $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$ has tangent vector $\mathbf{r}'_1(t) = \langle 1, 2t, 3t^2 \rangle$. At the origin, this tangent vector is $\mathbf{r}'_1(0) = \langle 1, 0, 0 \rangle$.

The curve $\mathbf{r}_2(t) = \langle \sin(t), \sin(2t), t \rangle$ has tangent vector $\mathbf{r}'_2(t) = \langle \cos(t), 2\cos(2t), 1 \rangle$. At the origin, this tangent vector is $\mathbf{r}'_2(0) = \langle 1, 2, 1 \rangle$.

The angle, $\theta$, between the two tangent vectors is given by

$$\cos(\theta) = \frac{\mathbf{r}'_1(0) \cdot \mathbf{r}'_2(0)}{|\mathbf{r}'_1(0)||\mathbf{r}'_2(0)|} = \frac{\sqrt{6}}{6}.$$ 

Thus

$$\theta = \arccos\left(\frac{\sqrt{6}}{6}\right) \approx 66^\circ.$$ 

29.

$$\int (16t^3 \mathbf{i} - 9t^2 \mathbf{j} + 25t^4 \mathbf{k}) \, dt = 4t^4 \mathbf{i} - 3t^3 \mathbf{j} + 5t^5 \mathbf{k} + \mathbf{C}.$$ 

31.

$$\int_0^{\pi/4} (\cos(2t) \mathbf{i} + \sin(2t) \mathbf{j} + t \sin(t) \mathbf{k}) \, dt$$

$$= \left( \frac{1}{2} \sin(2t) \mathbf{i} - \frac{1}{2} \cos(2t) \mathbf{j} + (\sin(t) - t \cos(t)) \mathbf{k} \right) \bigg|_{t=0}^{t=\pi/4}$$

$$= \left( \frac{1}{2} \mathbf{i} + \left( \frac{\sqrt{2}}{2} - \frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} \right) \mathbf{k} \right) - \left( \frac{1}{2} \mathbf{j} \right)$$

$$= \frac{1}{2} \mathbf{i} + \frac{1}{2} \mathbf{j} + \frac{\sqrt{2}}{2} \left( \frac{4 - \pi}{4} \right) \mathbf{k}.$$ 

33.

$$\int (e^t \mathbf{i} + 2t \mathbf{j} + \ln(t) \mathbf{k}) \, dt = e^t \mathbf{i} + t^2 \mathbf{j} + (t \ln(t) - t) \mathbf{k} + \mathbf{C}.$$
35. If \( r'(t) = t^2i + 4t^3j - t^2k \), then \( r(t) = \frac{1}{3}t^3i + t^4j - \frac{1}{3}t^3k + C \) where \( C \) is a constant vector. Since we must also have (according to the given information) that \( r(0) = j \), then we must have
\[
\frac{1}{3}(0)^3i + (0)^4j - \frac{1}{3}(0)^3k + C = j.
\]
This implies that \( C = j \) and hence that
\[
r(t) = \frac{1}{3}t^3i + (t^4 + 1)j - \frac{1}{3}t^3k.
\]

37. Let \( u(t) = \langle f_1(t), f_2(t), f_3(t) \rangle \) and let \( v(t) = \langle g_1(t), g_2(t), g_3(t) \rangle \). Then
\[
u(t) + v(t) = \langle f_1(t) + g_1(t), f_2(t) + g_2(t), f_3(t) + g_3(t) \rangle,
\]
\[
u'(t) = \langle f'_1(t), f'_2(t), f'_3(t) \rangle
\]
and hence that
\[
v'(t) = \langle g'_1(t), g'_2(t), g'_3(t) \rangle.
\]
Thus
\[
\frac{d}{dt}(u(t) + v(t)) = \langle f'_1(t) + g'_1(t), f'_2(t) + g'_2(t), f'_3(t) + g'_3(t) \rangle
\]
and
\[
\frac{d}{dt}(u(t) + v(t)) = u'(t) + v'(t).
\]

39. Let \( u(t) = \langle f_1(t), f_2(t), f_3(t) \rangle \) and let \( v(t) = \langle g_1(t), g_2(t), g_3(t) \rangle \). To make notation less cumbersome, we will suppress the \( t \)s that occur everywhere.
Thus
\[
u' = \langle f'_1, f'_2, f'_3 \rangle,
\]
and
\[
v'(t) = \langle g'_1, g'_2, g'_3 \rangle,
\]
and
\[
u(t) \times v(t) = \begin{vmatrix} i & j & k \\ f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{vmatrix}
= (f_2g_3 - f_3g_2)i - (f_1g_3 - f_3g_1)j + (f_1g_2 - f_2g_1)k.
\]
Thus,
\[
\frac{d}{dt}(u(t) \times v(t)) = (f_2g'_3 + f'_2g_3 - f_3g'_2 - f'_3g_2)i
- (f'_1g_3 + f'_1g_3 - f_3g'_1 - f'_3g_1)j
+ (f'_1g'_2 + f'_1g_2 - f_2g'_1 - f'_2g_1)k.
\]
Also,
\[ \mathbf{u}(t) \times \mathbf{v}'(t) = \begin{vmatrix} i & j & k \\ f_1 & f_2 & f_3 \\ g'_1 & g'_2 & g'_3 \end{vmatrix} = (f_2g'_3-f_3g'_2)i - (f_1g'_3-f_3g'_1)j + (f_1g'_2-f_2g'_1)k \]

and

\[ \mathbf{u}'(t) \times \mathbf{v}(t) = \begin{vmatrix} i & j & k \\ f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{vmatrix} = (f_2g_3-f_3g_2)i - (f_1g_3-f_3g_1)j + (f_1g_2-f_2g_1)k \]

so

\[ \mathbf{u}(t) \times \mathbf{v}'(t) + \mathbf{u}'(t) \times \mathbf{v}(t) = (f_2g'_3-f_3g'_2+f_2g_3-f_3g_2)i - (f_1g'_3-f_3g'_1+f_1g_3-f_3g_1)j + (f_1g'_2-f_2g'_1+f_1g_2-f_2g_1)k \]

which shows that

\[ \frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t)) = \mathbf{u}(t) \times \mathbf{v}'(t) + \mathbf{u}'(t) \times \mathbf{v}(t). \]

41. For \( \mathbf{u}(t) = i - 2t^2j + 3t^3k \) and \( \mathbf{v}(t) = ti + \cos(t)j + \sin(t)k \), we have

\[
\mathbf{u}'(t) = -4j + 9t^2k
\]

\[
\mathbf{v}'(t) = i - \sin(t)j + \cos(t)k.
\]

Thus

\[
\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t)) = \mathbf{u}(t) \cdot \mathbf{v}'(t) + \mathbf{u}'(t) \cdot \mathbf{v}(t)
\]

\[
= (1 + 2t^2 \sin(t) + 3t^3 \cos(t))
\]

\[
+ (-4t \cos(t) + 9t^2 \sin(t))
\]

\[
= 1 + 11t^2 \sin(t) + 3t^3 \cos(t) - 4t \cos(t).
\]

43.

\[
\frac{d}{dt}(\mathbf{r}(t) \times \mathbf{r}'(t)) = \mathbf{r}(t) \times \mathbf{r}''(t) + \mathbf{r}'(t) \times \mathbf{r}'(t).
\]

However, recall that if \( \mathbf{a} \) is any vector, then \( \mathbf{a} \times \mathbf{a} = \mathbf{0} \). Thus \( \mathbf{r}'(t) \times \mathbf{r}'(t) = \mathbf{0} \)

and therefore

\[
\frac{d}{dt}(\mathbf{r}(t) \times \mathbf{r}'(t)) = \mathbf{r}(t) \times \mathbf{r}''(t).
\]