Answers and Solutions for Section 11.1 Homework
Problems 1, 5, 7, 9, 15, 17, 19, 21, 31-36, 37, and 39
(A few of the solutions given here are for questions that are different than those asked in the book.)

1. (a) \( f(8, 60) = -7 \). This means that if the temperature is 8°C and the wind is blowing at 60 km/hr, then it feels like the temperature is about \(-7^\circ C\).

(b) The value of \( v \) such that \( f(-12, v) = -26 \) is the wind speed at which it feels like it is \(-26^\circ C\) when it is really \(-12^\circ C\). According to the given table, this value of \( v \) is 20 km/hr.

5. **Note:** It is not really correct to write down a formula and then ask what the domain of the function defined by this formula is. Specifying the domain is a part of defining a function. Nonetheless, it seems that all modern calculus textbooks ask this question. (Many mathematicians complain about it but, for some reason, the textbooks continue to do this.) It is OK, however, to write down a formula and then ask “What is the largest possible set on which this formula defines a function?” We will take this to be the meaning of “What is the domain?” in this and all future textbook problems that ask this question.

That having been said, the largest possible set that can be used as a domain in defining a function by the formula \( f(x, y) = \ln(9 - x^2 - 9y^2) \) is the set of all points, \((x, y)\), in \( \mathbb{R}^2 \) such that \( 9 - x^2 - 9y^2 > 0 \). We can write this condition as
\[
\frac{x^2}{9} + y^2 < 1.
\]
This set is the interior of an ellipse.

7. We are given the formula \( f(x, y, z) = \exp\left(\sqrt{z - x^2 - y^2}\right) \).

(a) \( f(2, -1, 6) = \exp\left(\sqrt{6 - 2^2 - (-1)^2}\right) = e. \)

(b) The largest possible set that can be used as a domain in defining a function by the formula
\[
f(x, y) = \exp\left(\sqrt{z - x^2 - y^2}\right)
\]
is the set of all points, \((x, y)\), in \( \mathbb{R}^2 \) such that \( z - x^2 - y^2 \geq 0 \). This condition can be written as
\[
x^2 + y^2 \leq z.
\]
This set is a paraboloid and all points that lie above the paraboloid. *(Note: Since \( f \) is a function of three variables, the domain of \( f \) is a subset of \( \mathbb{R}^3 \).)*
(c) Since $\sqrt{z - x^2 - y^2} \geq 0$, then $\exp\left(\sqrt{z - x^2 - y^2}\right) \geq 1$. We see that the range of $f$ is $[1, \infty)$.

9. It looks like $f(-3, 3) \approx 55$ and $f(3, -2) \approx 35$. The graph of $f$ is a hill that is steeper on one side than it is on the other.

15. The contours of $f(x, y) = xy$ are curves of the form $xy = k$. This is a family of hyperbolas.

17. The contours of $f(x, y) = y - \ln(x)$ are curves of the form $y - \ln(x) = k$ or $y = \ln(x) + k$. 

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19. The contours of \( f(x, y) = \sqrt{x+y} \) are curves of the form \( \sqrt{x+y} = k \) or \( x + y = k^2 \) or \( y = -x + k^2 \). This is a family of lines – all of which have slope 1 and all of which lie above the line \( y = -x \) in the \( xy \) plane.

21. The level curves of \( f(x, y) = x - y^2 \) are parabolas.

31. C and II

32. C and IV

33. F and I

34. E and III

35. B and VI

36. D and V

37. The level surfaces of \( f(x, y, z) = x + 3y + 5z \) are surfaces of the form \( x + 3y + 5z = k \). These are planes.

39. The level surfaces of \( f(x, y, z) = x^2 - y^2 + z^2 \) are surfaces of the form \( x^2 - y^2 + z^2 = k \). These are hyperboloids of one or two sheets.