1. \((4, 0, -3)\)

3. The point \(R(0, 3, 8)\) lies in the \(yz\) plane. The point \(P(6, 2, 3)\) has a distance of 2 from the \(xz\) plane. The point \(Q(-5, -1, 4)\) has a distance of 1 from the \(xz\) plane. The point \(R(0, 3, 8)\) has a distance of 3 from the \(xz\) plane. Thus, \(Q\) is closest to the \(xz\) plane.

5. The surface \(x + y = 2\) in \(\mathbb{R}^3\) is a plane that is perpendicular to the \(xy\) plane.

7. (a) 

\[
|PQ| = \sqrt{(7-3)^2 + (0-(-2))^2 + (1-(-3))^2} = 6
\]

\[
|QR| = \sqrt{(7-1)^2 + (0-2)^2 + (1-1)^2} = 2\sqrt{10} \approx 6.3246.
\]

\[
|RP| = \sqrt{(1-3)^2 + (2-(-2))^2 + (1-(-3))^2} = 6.
\]

This is an isosceles triangle (because two sides have the same length) but not a right triangle because

\[
|PQ|^2 + |RP|^2 = 6^2 + 6^2 = 72
\]

but

\[
|QR|^2 = \left(2\sqrt{10}\right)^2 = 40.
\]

(b)

\[
|PQ| = \sqrt{(4-2)^2 + (1-(-1))^2 + (1-0)^2} = 3
\]

\[
|QR| = \sqrt{(4-4)^2 + (1-(-5))^2 + (1-4)^2} = 3\sqrt{5} \approx 6.7082
\]

\[
|RP| = \sqrt{(2-(-4))^2 + (-1-(-5))^2 + (0-4)^2} = 6
\]

This is not an isosceles triangle (because no two sides have the same length) but is a right triangle because

\[
|PQ|^2 + |RP|^2 = 3^2 + 6^2 = 45 = \left(3\sqrt{5}\right)^2 = |QR|^2.
\]

9. (a) Note that \(|AB| = \sqrt{26}, |BC| = \sqrt{45}, \text{ and } |AC| = \sqrt{3}|. Since no two of these lengths add up to the other length, these points do not lie on the same line.
(b) Note that $|DE| = \sqrt{11}$, $|EF| = 2\sqrt{11}$, and $|DF| = 3\sqrt{11}$. Since $|DE| + |EF| = |DF|$, the points $D$, $E$, and $F$ must lie on the same line.

11. The radius of the sphere in question is

$$r = \sqrt{(4 - 3)^2 + (3 - 8)^2 + (-1 - 1)^2} = \sqrt{30}.$$

An equation for this sphere is

$$(x - 3)^2 + (y - 8)^2 + (z - 1)^2 = 30.$$

13. $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$ can be written (by using the method of completing the square) as

$$x^2 - 6x + 9 + y^2 + 4y + 4 + z^2 - 2z + 1 = 11 + 9 + 4 + 1$$

or as

$$(x - 3)^2 + (y + 2)^2 + (z - 1)^2 = 5^2.$$

This is an equation for the sphere with center at $(3, -2, 1)$ and radius $r = 5$.

15. Consider the points $P_1 (x_1, y_1, z_1)$, $P_2 (x_2, y_2, z_2)$, and $M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$.

To prove that $M$ is the midpoint of the line segment $P_1P_2$, we will show that $P_1$, $P_2$, and $M$ all lie on the same line and that $|P_1M| = |P_2M|$.

Note that

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

and also note that

$$|P_1M| = \sqrt{\left( \frac{x_1 + x_2}{2} - x_1 \right)^2 + \left( \frac{y_1 + y_2}{2} - y_1 \right)^2 + \left( \frac{z_1 + z_2}{2} - z_1 \right)^2}$$

$$= \sqrt{\left( \frac{x_2 - x_1}{2} \right)^2 + \left( \frac{y_2 - y_1}{2} \right)^2 + \left( \frac{z_2 - z_1}{2} \right)^2}$$

$$= \frac{1}{2} |P_1P_2|$$

It can be shown in a similar way that

$$|P_2M| = \frac{1}{2} |P_1P_2|.$$
Thus
\[ |P_1P_2| = |P_1M| + |P_2M| \]
which means that \( P_1, P_2, \) and \( M \) all lie on the same line. In addition, \( |P_1M| = |P_2M| \). Therefore, \( M \) is the midpoint of the line segment \( P_1P_2 \).

Now (as part b of this problem), we will find the lengths of the medians of the triangle with vertices at \( A(1, 2, 3), B(-2, 0, 5), \) and \( C(4, 1, 5) \). The midpoint of \( AB \) is
\[ M_{AB} = \left( \frac{1 + (-2)}{2}, \frac{2 + 0}{2}, \frac{3 + 5}{2} \right) = \left( -\frac{1}{2}, 1, 4 \right). \]
Thus the length of the line segment joining \( C \) to \( M_{AB} \) is
\[ |CM_{AB}| = \sqrt{\left( -\frac{1}{2} - 4 \right)^2 + (1 - 1)^2 + (4 - 5)^2} = \sqrt{\frac{85}{4}} = \frac{\sqrt{85}}{2}. \]
The lengths of the other two medians are found in a similar way.

17. If a sphere has center \( (2, -3, 6) \) and this sphere touches the \( xy \) plane at just one point, then this sphere must have radius 6. An equation for this sphere is
\[ (x - 2)^2 + (y + 3)^2 + (z - 6) = 36. \]
(Parts b and c of this exercise are done in a similar way.)

19. The equation \( y = -4 \) describes a plane in \( \mathbb{R}^3 \) that is parallel to the \( xz \) plane.

21. The inequality \( x > 3 \) describes a “half-space” in \( \mathbb{R}^3 \) that consists of all points in \( \mathbb{R}^3 \) that lie “in front of” the plane \( x = 3 \).

23. The inequality \( 0 \leq z \leq 6 \) describes the three dimensional “strip” of points that lie between the planes \( z = 0 \) and \( z = 6 \).

25. The inequality \( x^2 + y^2 + z^2 \leq 3 \) describes the set of all points that lie inside or on the sphere \( x^2 + y^2 + z^2 = 3 \).

27. The inequality \( x^2 + z^2 \leq 9 \) describes a solid cylinder of radius 3 whose central axis is the \( y \) axis.

29. \( y < 0 \).

31. \( r^2 < x^2 + y^2 + z^2 < R^2 \).