1. (a) scalar  
   (b) vector  
   (c) vector  
   (d) scalar

2. The point \((4, 7)\) is a point (with a definite position) in \(\mathbb{R}^2\). The vector \((4, 7)\) is an equivalence class of “arrows” that have length \(\sqrt{4^2 + 7^2} = \sqrt{65} \approx 8.06\) and direction that makes an angle of \(\arctan(7/4) \approx 60.26^\circ\) with the positive \(x\) axis. When we say that the vector is an “equivalence class”, we mean that the vector can be visualized as being positioned anywhere in \(\mathbb{R}^2\). A particular representative of the vector \((4, 7)\) is the arrow with base at the point \((0, 0)\) and head at the point \((4, 7)\). This particular representative of \((4, 7)\) is referred to as the position vector of the point \((4, 7)\). See the figure below. In this figure, all of the pictured arrow are representatives of the vector \((4, 7)\). The red arrow is the position vector of the point \((4, 7)\).
3. \( DA = CB, AB = DC, DE = EB, CE = EA. \)

Since the figure is a parallelogram, it is clear that \( AB = DC \) and that \( DA = CB \). It is somewhat less obvious that \( DE = EB \) and that \( CE = EA \). Here is how to prove the latter two equalities: It can be seen that

\[
DE = DA + xAC
\]

where \( x \) is some (unknown at this point) scalar with \( 0 < x < 1 \). Likewise,

\[
CE = CB + yBD
\]

where \( y \) is some scalar with \( 0 < y < 1 \). Since \( DA = CB \), we now have

\[
DE - xAC = CE - yBD.
\]

By subtracting the vector \( CE \) from both sides of the above equation, we obtain

\[
DC - xAC = -yBD
\]

which is the same as

\[
DC - x(AB + BC) = -y(BC + CD)
\]

or

\[
DC - x(AB - CB) = -y(-CB - DC).
\]

However, \( DC = AB \) and \( DA = CB \) so we have

\[
DC - x(DC - DA) = -y(-DA - DC),
\]

which gives us

\[
(1 - x - y)DC + (x - y)DA = 0.
\]
Since the vectors $DC$ and $DA$ are not parallel and the above equation says that a linear combination of these two vectors is equal to the zero vector, then both of the coefficients in this linear combination must be equal to zero. That is,

$$1 - x - y = 0$$

and

$$x - y = 0.$$

The latter equation tells us that $x = y$. Substituting this into the first equation gives $1 - x - x = 0$ and this implies that $x = 1/2$ and also that $y = 1/2$.

Now that we know that $x$ and $y$ are both equal to $1/2$, we have

$$AE = \frac{1}{2}AC$$

and

$$EC = \frac{1}{2}AC.$$

Hence $AE = EC$ (or equivalently, $EA = CE$). Likewise, $DE = EB$.

5. See the pictures below.

7. For $A(2, 3)$ and $B(-2, 1)$, we have $a = AB = (-2 - 2, 1 - 3) = (-4, -2)$.

9. $a = AB = (2 - 0, 3 - 3, -1 - 1) = (2, 0, -2)$. Two representatives of this vector (one of which is the position vector of the point $(2, 0, -2)$) are shown below.
11. \(\langle 3, -1 \rangle + \langle -2, 4 \rangle = \langle 1, 3 \rangle\)

13. \(\langle 0, 1, 2 \rangle + \langle 0, 0, -3 \rangle = \langle 0, 1, -1 \rangle.\)

15. (This is not the same problem that is in the book but is certainly similar.)
For \( \mathbf{a} = \langle -4, 3 \rangle \) and \( \mathbf{b} = \langle 6, 2 \rangle \), we have

\[
|\mathbf{a}| = \sqrt{(6 + 4)^2 + (2 - 3)^2} = \sqrt{101}
\]

\[
\mathbf{a} + \mathbf{b} = \langle -4 + 6, 3 + 2 \rangle = \langle 2, 5 \rangle
\]

\[
\mathbf{a} - \mathbf{b} = \langle -4 - 6, 3 - 2 \rangle = \langle -10, 1 \rangle
\]

\[
2\mathbf{a} = \langle 2(-4), 2(3) \rangle = \langle -8, 6 \rangle
\]

\[
3\mathbf{a} + 4\mathbf{b} = \langle 3(-4), 3(3) \rangle + \langle 4(6), 4(2) \rangle
\]

\[
= \langle -12, 9 \rangle + \langle 24, 8 \rangle
\]

\[
= \langle -12 + 24, 9 + 8 \rangle
\]

\[
= \langle 12, 17 \rangle
\]

17. (This is not the same problem that is in the book but is certainly similar.)
For \( \mathbf{a} = \mathbf{i} - \mathbf{2j} + \mathbf{k} \) and \( \mathbf{b} = \mathbf{j} + \mathbf{2k} \), we have

\[
|\mathbf{a}| = \sqrt{(0 - 1)^2 + (1 + 2)^2 + (2 - 1)^2} = \sqrt{11}
\]

\[
\mathbf{a} + \mathbf{b} = (1 + 0)\mathbf{i} + (-2 + 1)\mathbf{j} + (1 + 2)\mathbf{k} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}
\]

\[
\mathbf{a} - \mathbf{b} = (1 - 0)\mathbf{i} + (-2 - 1)\mathbf{j} + (1 - 2)\mathbf{k} = \mathbf{i} - 3\mathbf{j} - \mathbf{k}
\]

\[
2\mathbf{a} = 2(1)\mathbf{i} + 2(-2)\mathbf{j} + 2(1)\mathbf{k} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}
\]

\[
3\mathbf{a} + 4\mathbf{b} = 3(1)\mathbf{i} + 3(-2)\mathbf{j} + 3(1)\mathbf{k} + 4(1)\mathbf{j} + 4(2)\mathbf{k}
\]

\[
= 3\mathbf{i} - 6\mathbf{j} + 3\mathbf{k} + 4\mathbf{j} + 8\mathbf{k}
\]

\[
= 3\mathbf{i} - 2\mathbf{j} + 11\mathbf{k}
\]

19. If \( \mathbf{a} = 8\mathbf{i} - \mathbf{j} + 4\mathbf{k} \), then \( |\mathbf{a}| = \sqrt{8^2 + (-1)^2 + 4^2} = 9 \), so a unit vector with the same direction as \( \mathbf{a} \) is

\[
\mathbf{u} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{8}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}.
\]

21. Referring to the figure below, we see that if \( \mathbf{v} = x\mathbf{i} + y\mathbf{j} \), then \( x = 4 \cos(\pi/3) = 2 \) and \( y = 4 \sin(\pi/3) = 2\sqrt{3} \). Thus \( \mathbf{v} = 2\mathbf{i} + 2\sqrt{3}\mathbf{j} \).
23. Here we have

\[ F_1 = -10 \cos(45^\circ) \mathbf{i} + 10 \sin(45^\circ) \mathbf{j} = -5\sqrt{2} \mathbf{i} + 5\sqrt{2} \mathbf{j} \]
\[ F_2 = 12 \cos(30^\circ) \mathbf{i} + 12 \sin(30^\circ) \mathbf{j} = 6\sqrt{3} \mathbf{i} + 6 \mathbf{j} \]

so

\[ \mathbf{F} = \left(6\sqrt{3} - 5\sqrt{2}\right) \mathbf{i} + \left(6 + 5\sqrt{2}\right) \mathbf{j} \approx 3.32 \mathbf{i} + 13.07 \mathbf{j}. \]

The magnitude of \( \mathbf{F} \) is

\[ |\mathbf{F}| = \sqrt{\left(6\sqrt{3} - 5\sqrt{2}\right)^2 + \left(6 + 5\sqrt{2}\right)^2} \]
\[ = \sqrt{244 + 60\left(\sqrt{2} - \sqrt{6}\right)} \approx 13.49 \]

and the angle \( \theta \) (in the figure) is

\[ \theta = \arctan \left(\frac{6 + 5\sqrt{2}}{6\sqrt{3} - 5\sqrt{2}}\right) \approx 75.75^\circ. \]

25. The woman’s velocity vector relative to the surface of the water is

\[ \mathbf{v} = -3 \mathbf{i} + 22 \mathbf{j}. \]

Her speed is thus

\[ |\mathbf{v}| = \sqrt{(-3)^2 + 22^2} \approx 22.2 \text{ miles per hour} \]

and since the angle of her velocity vector relative to due west is

\[ \theta = \arctan \left(\frac{22}{9}\right) \approx 82.2^\circ, \]

she is heading in the direction of 7.8° west of due north.
27. Referring to the diagram (with distances measured in meters), we have

\[ \mathbf{T}_1 = -|\mathbf{T}_1| \cos(1.15^\circ) \mathbf{i} + |\mathbf{T}_1| \sin(1.15^\circ) \mathbf{j} \]
\[ \mathbf{T}_2 = |\mathbf{T}_2| \cos(1.15^\circ) \mathbf{i} + |\mathbf{T}_2| \sin(1.15^\circ) \mathbf{j}. \]

Since the weight of the shirt is

\[ 0.8 \text{ kg} \times (9.8 \text{ m/s}^2) \approx 7.84 \text{ Newtons}, \]

we must have

\[ \mathbf{T}_1 + \mathbf{T}_2 - 7.84 \mathbf{j} = \mathbf{0} \]

or

\[ (|\mathbf{T}_2| - |\mathbf{T}_1|) \cos(1.15^\circ) \mathbf{i} + (|\mathbf{T}_2| + |\mathbf{T}_1|) \sin(1.15^\circ) \mathbf{j} - 7.84 \mathbf{j} = \mathbf{0}. \]

This implies that \(|\mathbf{T}_1| = |\mathbf{T}_2|\) (which is intuitively obvious) and also that

\[ 2 |\mathbf{T}_1| \sin(1.15^\circ) = 7.84 = 0. \]

Therefore,

\[ |\mathbf{T}_1| = 7.84/(2 \sin(1.15^\circ)) = 195.3 \text{ Newtons} \]

and the tensions are

\[ \mathbf{T}_1 = -195.3 \cos(1.15^\circ) \mathbf{i} + 195.3 \sin(1.15^\circ) \mathbf{j} \approx -195.3 \mathbf{i} + 3.92 \mathbf{j} \]
\[ \mathbf{T}_2 = 195.3 \cos(1.15^\circ) \mathbf{i} + 195.3 \sin(1.15^\circ) \mathbf{j} \approx 195.3 \mathbf{i} + 3.92 \mathbf{j}. \]
Referring to the figure above, we want to prove that $BD = \frac{1}{2}AE$. To do this, first note that, since $B$ is the midpoint of $AC$ and $D$ is the midpoint of $CE$, we have

$$2AB = AC$$
$$2CD = CE.$$

Also

$$AC + CE = AE$$

so we have

$$2AB + 2CD = AE$$

and

$$2(AB + CD) = AE$$

and

$$AB + CD = \frac{1}{2}AE.$$ 

Now note that

$$BC + CD = BD$$

but also note that $AB = BC$ and thus

$$AB + CD = BD.$$ 

This proves that $BD = \frac{1}{2}AE$. 

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