1. (a) \( \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \) is meaningful. It is a scalar.
   
   (b) \( \mathbf{a} \times (\mathbf{b} \cdot \mathbf{c}) \) is not meaningful. We can’t take the dot product of a vector and a scalar.
   
   (c) \( \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \) is meaningful. It is a vector.
   
   (d) \( (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c} \) is not meaningful.
   
   (e) \( (\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d}) \) is not meaningful. We can’t take the cross product of two scalars.
   
   (f) \( (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) \) is meaningful. It is a scalar.

2. 

\[
|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin(60^\circ) = (5)(10) \left( \frac{\sqrt{3}}{2} \right) = 25\sqrt{3}.
\]

\( \mathbf{u} \times \mathbf{v} \) is directed into the page.

3. 

\[
|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin(30^\circ) = (6)(8) \left( \frac{1}{2} \right) = 24.
\]

\( \mathbf{u} \times \mathbf{v} \) is directed into the page.

4. 

\[
|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin(90^\circ) = (3)(2)(1) = 6.
\]

The \( x \) component of \( \mathbf{a} \times \mathbf{b} \) is positive, the \( y \) component is negative, and \( z \) component is zero.

5. The magnitude of the torque is 

\[
|\tau| = (0.18)(60) \sin(80^\circ) \approx 10.64 \text{ Joules.}
\]

6. The distance from the axis of the bolt (P) to the point where the force is applied is \( 4\sqrt{2} \) and the force is applied at an angle of \( 105^\circ \) from the line along which the force is applied. Thus

\[
|\tau| = \left( 4\sqrt{2} \right)(36) \sin(105^\circ) \approx 197 \text{ ft-lb.}
\]

7. For \( \mathbf{a} = (1, 2, 0) \) and \( \mathbf{b} = (0, 3, 1) \), we have

\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 2 & 0 \\
0 & 3 & 1 \\
\end{vmatrix}
= ((2)(1) - (0)(3)) \mathbf{i} - ((1)(1) - (0)(0)) \mathbf{j} + ((1)(3) - (0)(2)) \mathbf{k}
= 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}.
\]
Note that

\[ a \cdot (a \times b) = (1)(2) + (2)(-1) + (0)(3) = 0 \]

and

\[ b \cdot (a \times b) = (0)(2) + (3)(-1) + (1)(3) = 0 \]

which shows that \( a \times b \) is orthogonal to both \( a \) and \( b \).

8. For \( a = \langle 5, 1, 4 \rangle \) and \( b = \langle -1, 0, 2 \rangle \), we have

\[
a \times b = \begin{vmatrix}
i & j & k \\
5 & 1 & 4 \\
-1 & 0 & 2 \\
\end{vmatrix}
= ((1)(2) - (4)(0))i - ((5)(2) - (4)(-1))j + ((5)(0) - (1)(-1))k
= 2i - 14j + k.
\]

Note that

\[ a \cdot (a \times b) = ((5)(2) - (1)(14)) + (4)(1) = 0 \]

and

\[ b \cdot (a \times b) = (-1)(2) + (0)(14) + (2)(1) = 0 \]

which shows that \( a \times b \) is orthogonal to both \( a \) and \( b \).

9. For \( a = \langle t, t^2, t^3 \rangle \) and \( b = \langle 1, 2t, 3t^2 \rangle \), we have

\[
a \times b = \begin{vmatrix}
i & j & k \\
t & t^2 & t^3 \\
1 & 2t & 3t^2 \\
\end{vmatrix}
= ((t^2)(3t^2) - (t^3)(2t))i - ((t)(3t^2) - (t^3)(1))j + ((t)(2t) - (t^2)(1))k
= t^4i - 2t^3j + t^2k.
\]

Note that

\[ a \cdot (a \times b) = (t)(t^4) + (t^2)(-2t^3) + (t^3)(t^2) = 0 \]

and

\[ b \cdot (a \times b) = (1)(t^4) + (2t)(-2t^3) + (3t^2)(t^2) = 0 \]

which shows that \( a \times b \) is orthogonal to both \( a \) and \( b \).

10. For \( a = i + e^tj + e^{-t}k \) and \( b = 2i + e^tj - e^{-t}k \), we have

\[
a \times b = \begin{vmatrix}
i & j & k \\
e^t & e^{-t} & 0 \\
1 & e^t & e^{-t} \\
\end{vmatrix}
= ((e^t)(-e^{-t}) - (e^{-t})(e^t))i - ((1)(-e^{-t}) - (e^t)(2))j + ((1)(e^t) - (e^{-t})(2))k
= -2i + 3e^{-t}j - e^t k.
\]
Note that
\[ \mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = (1) (-2) + (e^t) (3e^{-t}) + (e^{-t}) (-e^t) = 0 \]
and
\[ \mathbf{b} \cdot (\mathbf{a} \times \mathbf{b}) = (2) (-2) + (e^t) (3e^{-t}) + (-e^{-t}) (-e^t) = 0 \]
which shows that \( \mathbf{a} \times \mathbf{b} \) is orthogonal to both \( \mathbf{a} \) and \( \mathbf{b} \).

11. For \( \mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} \) and \( \mathbf{b} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} \), we have

\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & 2 & 4 \\
1 & -2 & -3
\end{vmatrix}
= ((3) (-3) - (4) (-2)) \mathbf{i} - ((3) (-3) - (4) (1)) \mathbf{j} + ((3) (-2) - (2) (1)) \mathbf{k}
= 2\mathbf{i} + 13\mathbf{j} - 8\mathbf{k}.
\]

Note that
\[ \mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = (3) (2) + (2) (13) + (4) (-8) = 0 \]
and
\[ \mathbf{b} \cdot (\mathbf{a} \times \mathbf{b}) = (1) (2) + (-2) (13) + (-3) (-8) = 0 \]
which shows that \( \mathbf{a} \times \mathbf{b} \) is orthogonal to both \( \mathbf{a} \) and \( \mathbf{b} \).

12. For \( \mathbf{a} = \mathbf{i} - 2\mathbf{k} \) and \( \mathbf{b} = \mathbf{j} + \mathbf{k} \), we have

\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 0 & -2 \\
0 & 1 & 1
\end{vmatrix}
= ((0) (1) - (-2) (1)) \mathbf{i} - ((1) (1) - (-2) (0)) \mathbf{j} + ((1) (1) - (0) (0)) \mathbf{k}
= 2\mathbf{i} - \mathbf{j} + \mathbf{k}.
\]

Note that
\[ \mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = (1) (2) + (0) (-1) + (-2) (1) = 0 \]
and
\[ \mathbf{b} \cdot (\mathbf{a} \times \mathbf{b}) = (0) (2) + (1) (-1) + (1) (1) = 0 \]
which shows that \( \mathbf{a} \times \mathbf{b} \) is orthogonal to both \( \mathbf{a} \) and \( \mathbf{b} \).

16. First note that
\[
\overrightarrow{KL} = (0, 1, 3) \\
\overrightarrow{KM} = (2, 6, 3) \\
\overrightarrow{KN} = (2, 5, 0) \\
\overrightarrow{LM} = (2, 5, 0) \\
\overrightarrow{LN} = (2, 4, -3) \\
\overrightarrow{MN} = (0, -1, -3)
\]

3
which shows that $MLKN$ is a parallelogram with opposite sides $\overrightarrow{LM} = \overrightarrow{KN}$ and $\overrightarrow{LK} = \overrightarrow{MN}$. The area of this parallelogram is

$$\overrightarrow{LK} \times \overrightarrow{LM} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & -3 \\ 2 & 5 & 0 \end{vmatrix}$$

$$= |(((-1)(0) - (-3)(5)) \mathbf{i} - ((0)(0) - (-3)(2)) \mathbf{j} + ((0)(5) - (-1)(2)) \mathbf{k}|$$

$$= |15\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}|$$

$$= \sqrt{15^2 + (-6)^2 + 2^2}$$

$$= \sqrt{759.415}$$

$$\approx 871.44.$$

As a remark, we observe that the parallelogram law holds:

$$|\overrightarrow{KM}|^2 + |\overrightarrow{LN}|^2 = 2|\overrightarrow{LK}|^2 + 2|\overrightarrow{LM}|.$$  

Check:

$$|\overrightarrow{KM}|^2 = 2^2 + 6^2 + 3^2 = 49$$

$$|\overrightarrow{LN}|^2 = 2^2 + 4^2 + (-3)^2 = 29$$

$$|\overrightarrow{LK}|^2 = 0^2 + 1^2 + 3^2 = 10$$

$$|\overrightarrow{LM}|^2 = 2^2 + 5^2 + 0^2 = 29$$

and

$$49 + 29 = 2(10) + 2(29).$$

20. Since $|\mathbf{u}| = 3$ and $|\mathbf{v}| = |5\mathbf{j}| = 5$, then

$$|\mathbf{u} \times \mathbf{v}| = 15 \sin(\theta)$$

where $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$ ($0^\circ \leq \theta \leq 180^\circ$). The minimum value of $|\mathbf{u} \times \mathbf{v}|$ is 0 (when $\theta = 0^\circ$ or $180^\circ$) and the maximum value of $|\mathbf{u} \times \mathbf{v}|$ is 15 (when $\theta = 90^\circ$). $\mathbf{u} \times \mathbf{v}$ points “out of the page” when $\mathbf{u}$ points into quadrant I or IV and points “into the page” when $\mathbf{u}$ points into quadrant II or III.

27. Referring to the diagram below
the distance from the point $P$ to the line $L$ is $d$ and it can be see that

$$d = |b| \sin (\theta).$$

By multiplying both sides of this equation by $|a|$, we obtain

$$|a| d = |a| |b| \sin (\theta).$$

Since

$$|a| |b| \sin (\theta) = |a \times b|,$$

we have

$$|a| d = |a \times b|$$

and this gives us

$$d = \frac{|a \times b|}{|a|}.$$