Answers and Solutions to Section 9.6 Homework Problems
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1. 
   a. \( f(40, 15) = 25 \). This means that when the wind has been blowing at
      40 knots for 15 hours, the wave height is 25 feet.
   b. \( h = f(30, t) \) gives the wave height as a function of time when the
      wind is blowing at 30 knots. It is an increasing function of \( t \).
   c. \( h = f(v, 30) \) gives the wave height as a function of wind speed when
      the wind has been blowing for 30 hours. It is an increasing function
      of \( v \).

3. For \( f(x, y) = x^2e^{3y} \):
   a. \( f(2, 0) = 2^2e^{3\cdot0} = 4 \)
   b. The (implied) domain of \( f \) is \( \mathbb{R}^2 \).
   c. The range of \( f \) is \( [0, \infty) \).

5. \( \{(x, y) \mid x + y \geq 0\} \). This is a half–plane (in \( \mathbb{R}^2 \)) lying on one side and
   including the line \( x + y = 0 \).

7. \( \{(x, y) \mid y \geq x^2 \text{ and } x \neq \pm 1\} \). This is all points on and above the parabola
   \( y = x^2 \) except those points on the vertical lines \( x = \pm 1 \).

9. a plane parallel to the \( xy \) plane
11. a plane
13. a parabolic cylinder
15. 
   a. VI
   b. V
   c. I
   d. IV
   e. II
   f. III
17. an elliptic paraboloid

19. a hyperbolic paraboloid

21. We can write $x = 4y^2 + z^2 - 4z + 4$ as $x = 4y^2 + (z - 2)^2$ or as

$$\frac{x}{4} = \frac{y^2}{1} + \frac{(z - 2)^2}{4}.$$ 

This is an equation of an elliptic paraboloid.

23. 
   a. In $\mathbb{R}^2$, the equation $x^2 + y^2 = 1$ is an equation representing a circle.
   b. In $\mathbb{R}^3$, the equation $x^2 + y^2 = 1$ is an equation representing a circular cylinder.
   c. The equation $x^2 + z^2 = 1$ is a circle in $\mathbb{R}^2$ and a circular cylinder in $\mathbb{R}^3$.

25. 
   a. For the equation $x^2 + y^2 - z^2 = 1$, if we set $x = k$, we obtain $y^2 - z^2 = 1 - k^2$ (an equation of a hyperbola assuming that $|k| < 1$). Likewise, if we set $y = k$, we obtain $x^2 - z^2 = 1 - k^2$ (an equation of a hyperbola assuming that $|k| < 1$). If we set $z = k$, we obtain $x^2 + y^2 = 1 + k^2$ (an equation of a circle).
   b. The hyperboloid of one sheet in part a has its central axis parallel to the $z$ axis. By similar reasoning, the hyperboloid of one sheet $x^2 - y^2 + z^2 = 1$ has its central axis parallel to the $y$ axis.
   c. The equation $x^2 + y^2 + 2y - z^2 = 0$ can be written as $x^2 + (y + 1)^2 - z^2 = 1$. This is a translation of the hyperboloid in part a by one unit in the negative $y$ direction.