3. (a) This point (represented by its position vector in the picture below) has cylindrical coordinates $(3, \pi/2, 1)$ and rectangular coordinates $(0, 3, 1)$. The point lies directly above the $y$ axis.

(b) This point (represented by its position vector in the picture below) has cylindrical coordinates $(4, -\pi/3, 5)$ and rectangular coordinates $(-2, -2\sqrt{3}, 5)$.

5. (a) If the rectangular coordinates of a point are $(x, y, z) = (1, -1, 4)$, then cylindrical coordinates of this point are $(r, \theta, z)$ where

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$
$$\tan(\theta) = \frac{y}{x} = -1 \quad \text{so} \quad \theta = -\frac{\pi}{4}$$
$$z = z = 4.$$

Thus, cylindrical coordinates for this point are $(r, \theta, z) = (\sqrt{2}, -\pi/4, 4)$.
(b) If the rectangular coordinates of a point are \((x, y, z) = (-1, -\sqrt{3}, 2)\), then cylindrical coordinates of this point are \((r, \theta, z)\) where

\[
r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2
\]

\[
\tan(\theta) = \frac{y}{x} = \sqrt{3} \quad \text{so} \quad \theta = -\frac{2\pi}{3}
\]

\[
z = z = 2.
\]

Thus, cylindrical coordinates for this point are \((r, \theta, z) = (2, -\frac{2\pi}{3}, 2)\).

7. (a) The point whose spherical coordinates are \((1, 0, 0)\) has rectangular coordinates \((0, 0, 1)\). This point lies on the \(z\) axis.

(b) The point whose spherical coordinates are \((2, \frac{\pi}{3}, \frac{\pi}{4})\) has rectangular coordinates \((\sqrt{2}/2, \sqrt{6}/2, \sqrt{2})\). The position vector of this point is shown in the figure below.

![Position Vector Figure](image)

9. (a) The point whose rectangular coordinates are \((x, y, z) = (-3, 0, 0)\) has spherical coordinates \((\rho, \theta, \phi) = (3, \pi, \pi/2)\).

(b) The point whose rectangular coordinates are \((x, y, z) = (0, 2, -2)\) has spherical coordinates \((\rho, \theta, \phi)\) where

\[
\rho = \sqrt{x^2 + y^2 + z^2} = 2\sqrt{2}
\]

\[
\cos(\phi) = \frac{z}{\rho} = \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2} \quad \text{so} \quad \phi = \frac{3\pi}{4}
\]

\[
\cos(\theta) = \frac{x}{\rho \sin(\phi)} = 0 \quad \text{so} \quad \theta = \frac{\pi}{2}
\]

Thus, spherical coordinates for this point are \((2\sqrt{2}, \pi/2, 3\pi/4)\).

11. Assuming that we are using cylindrical coordinates, \(r = 3\) is the equation of a cylinder of radius 3 whose central axis is the \(z\) axis.
13. Assuming that we are using spherical coordinates, \( \phi = \pi / 3 \) is the equation of the upper half of a cone. The vertex of this cone is at the origin and the central axis is the \( z \) axis. The angle between the cone and the \( z \) axis is \( \pi / 3 \) radians (or 60°).

15. The equation \( z = r^2 \) (in cylindrical coordinates) can be written in rectangular coordinates as \( z = x^2 + y^2 \). This is the equation of a paraboloid with vertex at the origin and with the \( z \) axis as its central axis.

17. The equation \( r = 2 \cos (\theta) \) (in cylindrical coordinates) can be written as \( r^2 = 2r \cos (\theta) \) and can then be written in rectangular coordinates as \( x^2 + y^2 = 2x \) or as \( (x - 1)^2 + y^2 = 1 \). This is the equation of a circular cylinder whose whose central axis is the line parallel to the \( z \) axis passing through the point \((1, 0, 0)\) (in rectangular coordinates). See the figure below.

19. The equation \( r^2 + z^2 = 25 \) (in cylindrical coordinates) can be written in rectangular coordinates as \( x^2 + y^2 + z^2 = 25 \). This is the equation of a sphere of radius 5 centered at the origin.

21. The given equation (in rectangular coordinates) is

\[ x^2 + y^2 + z^2 = 16. \]

The equivalent equation in cylindrical coordinates is

\[ r^2 + z^2 = 16. \]
The equivalent equation in spherical coordinates is
\[ \rho = 4. \]

23. The given equation (in rectangular coordinates) is
\[ x^2 + y^2 = 2y. \]
We can write this equation as
\[ x^2 + (y - 1)^2 = 1 \]
and hence see that it is the equation of a cylinder of radius 1 whose central axis is the line parallel to the z axis passing through the point (0, 1, 0).

The equivalent equation in cylindrical coordinates is
\[ r^2 = 2r \sin(\theta) \]
or simply
\[ r = 2 \sin(\theta). \]

The equivalent equation in spherical coordinates is
\[ \rho^2 \sin^2(\phi) \cos^2(\theta) + \rho^2 \sin^2(\phi) \sin^2(\theta) = 2\rho \sin(\phi) \sin(\theta). \]
This can be simplified to
\[ \rho \sin^2(\phi) = 2 \sin(\phi) \sin(\theta). \]

We can do a further simplification: Normally, it would be dangerous to divide both sides of the above equation by \( \sin(\phi) \), for what if \( \sin(\phi) = 0? \)
Well, if \( \sin(\phi) = 0 \), then we are on a point of the cylinder that lies right on the z axis. In this case it is also okay to assume that \( \sin(\theta) = 0 \).
Therefore, we can write the above equation as
\[ \rho \sin(\phi) = 2 \sin(\theta). \]