5. The atoms involved in the reaction are boron ($B$), sulphur ($S$), hydrogen ($H$), and oxygen ($O$).

Boron sulfide ($B_2S_3$) is represented by the vector

$$\begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}.$$

Water ($H_2O$) is represented by the vector

$$\begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$

Boric acid ($H_3BO_3$) is represented by the vector

$$\begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix},$$

and hydrogen sulfide ($H_2S$) is represented by the vector

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}.$$

The balance equation for this reaction can be written as

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}.$$

Another way to write this is
from which we can see that we have a homogeneous system with coefficient matrix

\[
A = \begin{bmatrix}
  2 & 0 & -1 & 0 \\
  3 & 0 & 0 & -1 \\
  0 & 2 & -3 & -2 \\
  0 & 1 & -3 & 0 \\
\end{bmatrix}.
\]

Since

\[
\text{rref}(A) = \begin{bmatrix}
  1 & 0 & 0 & -\frac{1}{3} \\
  0 & 1 & 0 & -2 \\
  0 & 0 & 1 & -\frac{2}{3} \\
  0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

we see that \(x_4\) is a free variable and that

\[
x_1 = \frac{1}{3}x_4, \\
x_2 = 2x_4, \\
x_3 = \frac{2}{3}x_4.
\]

In order to make all of the coefficients of the reaction be whole numbers, we choose \(x_4 = 3\). This shows us the correct balancing of the reaction equation is

\[
B_2S_3 + 6H_2O \rightarrow 2H_3BO_3 + 3H_2S.
\]

7. The atoms involved are sodium (\(Na\)), hydrogen (\(H\)), carbon (\(C\)), and oxygen (\(O\)).

Sodium bicarbonate (\(NaHCO_3\)) is represented by the vector

\[
\begin{bmatrix}
  1 \\
  1 \\
  1 \\
  3 \\
\end{bmatrix}.
\]

Citric acid (\(H_3C_6H_5O_7\)) is represented by the vector
Sodium citrate \((Na_3C_6H_5O_7)\) is represented by the vector
\[
\begin{bmatrix}
3 \\
5 \\
6 \\
7
\end{bmatrix}
\]

Water \((H_2O)\) is represented by the vector
\[
\begin{bmatrix}
0 \\
2 \\
0 \\
1
\end{bmatrix}
\]

Carbon dioxide \((CO_2)\) is represented by the vector
\[
\begin{bmatrix}
0 \\
0 \\
1 \\
2
\end{bmatrix}
\]

The balance equation for this reaction can be written as
\[
x_1 \begin{bmatrix}
1 \\
1 \\
3
\end{bmatrix} + x_2 \begin{bmatrix}
0 \\
8 \\
6 \\
7
\end{bmatrix} = x_3 \begin{bmatrix}
3 \\
5 \\
6 \\
7
\end{bmatrix} + x_4 \begin{bmatrix}
0 \\
2 \\
0 \\
1
\end{bmatrix} + x_5 \begin{bmatrix}
0 \\
0 \\
1 \\
2
\end{bmatrix}
\]

Another way to write this is
\[
x_1 \begin{bmatrix}
1 \\
1 \\
3
\end{bmatrix} + x_2 \begin{bmatrix}
0 \\
8 \\
6 \\
7
\end{bmatrix} - x_3 \begin{bmatrix}
3 \\
5 \\
6 \\
7
\end{bmatrix} - x_4 \begin{bmatrix}
0 \\
2 \\
0 \\
1
\end{bmatrix} - x_5 \begin{bmatrix}
0 \\
0 \\
1 \\
2
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

from which we can see that we have a homogeneous system with coefficient matrix
\[ A = \begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 \\ 1 & 6 & -6 & 0 & -1 \\ 3 & 7 & -7 & -1 & -2 \end{bmatrix} . \]

Since
\[ \text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} , \]

we see that \( x_5 \) is a free variable and that
\[
\begin{align*}
x_1 &= x_5 \\
x_2 &= \frac{1}{3} x_5 \\
x_3 &= \frac{1}{3} x_5 \\
x_4 &= x_5
\end{align*}
\]

In order to make all of the coefficients of the reaction be whole numbers, we choose \( x_5 = 3 \). This shows us the correct balancing of the reaction equation is
\[
3\text{NaHCO}_3 + H_3\text{C}_6\text{H}_5\text{O}_7 \rightarrow \text{Na}_3\text{C}_6\text{H}_5\text{O}_7 + 3\text{H}_2\text{O} + 3\text{CO}_2 .
\]

11. Traffic flow through the junction \( A \) is described by the equation
\[ x_1 + x_3 = 20 . \]

Traffic flow through the junction \( B \) is described by the equation
\[ x_2 = x_3 + x_4 . \]

Traffic flow through the junction \( C \) is described by the equation
\[ 80 = x_1 + x_2 . \]

(I am assuming that the arrow on the road with flow 80 should be pointing in the opposite direction in order for the diagram to make sense.)

We thus have a system of linear equations
\[
\begin{align*}
x_1 + x_3 &= 20 \\
x_2 - x_3 - x_4 &= 0 \\
x_1 + x_2 &= 80
\end{align*}
\]

whose augmented matrix is
which is equivalent to the reduced row echelon matrix
\[
\begin{bmatrix}
1 & 0 & 1 & 0 & 20 \\
0 & 1 & -1 & -1 & 0 \\
1 & 1 & 0 & 0 & 80
\end{bmatrix}
\]
We now see that the general solution of our system is
\[
x_1 = 20 - x_3 \\
x_2 = 60 + x_3 \\
x_3 = \text{free} \\
x_4 = 60.
\]
Since all traffic flows are assumed to be nonnegative, the largest possible value for \(x_3\) is 20 (for if \(x_3 > 20\), then \(x_1 < 0\)).

13. The network pictured is described by the linear system
\[
-x_1 + x_2 = 50 \\
-x_2 + x_3 - x_4 + x_5 = 0 \\
-x_5 + x_6 = -60 \\
x_4 - x_6 = 50 \\
x_1 - x_3 = -40
\]
whose augmented matrix is
\[
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 0 & 50 \\
0 & -1 & 1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & -60 \\
0 & 0 & 0 & 1 & 0 & -1 & 50 \\
1 & 0 & -1 & 0 & 0 & 0 & -40
\end{bmatrix}
\]
This matrix is equivalent to
\[
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & -40 \\
0 & 1 & -1 & 0 & 0 & 0 & 10 \\
0 & 0 & 0 & 1 & 0 & -1 & 50 \\
0 & 0 & 0 & 0 & 1 & -1 & 60 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
The general flow of this traffic flow network is thus
\begin{align*}
x_1 &= -40 + x_3 \\
x_2 &= 10 + x_3 \\
x_3 &= \text{free} \\
x_4 &= 50 + x_6 \\
x_5 &= 60 + x_6 \\
x_6 &= \text{free}.
\end{align*}

If we want all flows to be nonnegative, then we must have \(x_3\) be at least 40. If \(x_3 = 40\), then we have
\begin{align*}
x_1 &= 0 \\
x_2 &= 50 \\
x_3 &= 40.
\end{align*}

It is also possible to have \(x_6 = 0\), in which case we would have
\begin{align*}
x_4 &= 50 \\
x_5 &= 60.
\end{align*}