Solutions to Selected Section 3.1 Homework Problems
Problems 1–13 (odd) and 19–40 (all)
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5.

\[
\begin{vmatrix}
2 & 3 & -4 \\
4 & 0 & 5 \\
5 & 1 & 6
\end{vmatrix}
= 2
\begin{vmatrix}
0 & 5 \\
1 & 6
\end{vmatrix}
- 3
\begin{vmatrix}
4 & 5 \\
5 & 6
\end{vmatrix}
+ (-4)
\begin{vmatrix}
4 & 0 \\
5 & 1
\end{vmatrix}
= -23.
\]

11.

\[
\begin{vmatrix}
3 & 5 & -8 & 4 \\
0 & -2 & 3 & -7 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 2
\end{vmatrix}
= 2
\begin{vmatrix}
3 & 5 & -8 \\
0 & -2 & 3 \\
0 & 0 & 1
\end{vmatrix}
= 2(1)
\begin{vmatrix}
3 & 5 \\
0 & -2
\end{vmatrix}
= 2(1)(-6)
= -12.
\]

21. If

\[
A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix} 3 & 4 \\ 5 + 3k & 6 + 4k \end{bmatrix},
\]

then \( B \) is obtained from \( A \) by a row replacement operation, so \( \det(B) = \det(A) \).

25. Since

\[
E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{bmatrix}
\]

is an elementary matrix that is obtained by performing a row replacement operation in \( I_3 \), we see that \( \det(E) = 1 \).
31. The determinant of an elementary row replacement matrix is 1.

33. For
\[ E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \]
we have
\[ EA = \begin{bmatrix} c & d \\ a & b \end{bmatrix}. \]

Thus
\[ \det (EA) = cb - da \]
\[ \det (E) = -1 \]
\[ \det (A) = ad - bc \]

and we observe that
\[ \det (EA) = cb - da = -(ad - bc) = -1 \cdot \det (A) = \det (E) \det (A). \]

37. For
\[ A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}, \]
we have
\[ 5A = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}. \]

Observe that
\[ \det (5A) = (15) (10) - 5 (20) = 50 \]

and
\[ \det (A) = 2. \]

Thus \( \det (5A) \neq 5 \det (A) \). In fact, since \( 5A \) is obtained from \( A \) by performing two scaling operations (scaling both rows of \( A \) by a factor of 5), we see that \( \det (5A) = 5^2 \cdot \det (A) \). In general, if \( A \) is an \( n \times n \) matrix and \( k \) is a scalar, then \( \det (kA) = k^n \cdot \det (A) \).

39. (a) An \( n \times n \) determinant is defined by determinants of \( (n - 1) \times (n - 1) \) submatrices. True.
(b) The \((i, j)\) cofactor of a matrix \(A\) is the matrix \(A_{ij}\) obtained from \(A\) by deleting its \(i\)th row and \(j\)th column. False. The \((i, j)\) cofactor of a matrix \(A\) is the matrix \(C_{ij} = (-1)^{i+j} \det (A_{ij})\) where \(A_{ij}\) is the matrix obtained from \(A\) by deleting its \(i\)th row and \(j\)th column.

40. (a) The cofactor expansion of \(\det (A)\) down a column is the negative of the cofactor expansion of \(\det(A)\) along a row. False. The cofactor expansion of \(\det (A)\) down a column is the same as the cofactor expansion of \(\det(A)\) along a row.

(b) The determinant of a triangular matrix is the sum of the entries on the main diagonal. False. The determinant of a triangular matrix is the product of the entries on the main diagonal.