1. Consider the autonomous differential equation
\[ \frac{dy}{dt} = (y - 3) (y^2 - 4). \]

(a) Find the equilibrium points of this differential equation.

(b) Construct the phase line for this differential equation, including a justification of why the phase line looks as it does (why arrows point in the direction they do, etc.)

(c) Draw the graph (in the \( t, y \) plane) of the particular solution of the above differential equation that satisfies the condition \( y(0) = 2.5 \). (Make sure to indicate any horizontal asymptotes that the graph has.)

2. Repeat problem 1 for the differential equation
\[ \frac{dy}{dt} = (y + 3) (y^2 - 4). \]

3. Consider the parameterized family of differential equations
\[ \frac{dy}{dt} = (y + \alpha) (y^2 - 4) \]
where the parameter \( \alpha \) is allowed to be any real number.

(a) Determine the bifurcation values(s) for this family of differential equations.

(b) Sketch phase lines that correspond to values of \( \alpha \) that are slightly less and slightly greater than the bifurcation value(s).

(c) Draw the bifurcation diagram (in the \( \alpha, y \) plane) for this family. (The bifurcation diagram must include solid curves for sinks and dashed curves for sources and you should include a discussion of how you determine whether the equilibria are sinks or sources.)

4. Show in detail how to solve the initial value problem
\[ \frac{dy}{dt} = -2ty + 4e^{-t^2}, \quad y(0) = 0. \]
5. A 100 gallon tank initially contains 100 gallons of saltwater with a concentration of 0.5 pounds of salt per gallon. There is a stirrer in the tank that keeps the saltwater well-mixed at all times. Pure water is added to the tank at the rate of 2 gallons per minute while the saltwater mixture is allowed to drain from the tank at the rate of 3 gallons per minute. Answer each of the following questions by giving detailed explanations (mathematical and written details – not just answers).

(a) How long will it take for the tank to drain completely?

(b) Let $y(t)$ be the amount (in pounds) of salt in the tank at time $t$ and let $c(t)$ be the concentration (in pounds per gallon) of salt in the tank at time $t$. Find formulas for $y(t)$ and $c(t)$.

(c) What is the concentration of the salt in the tank at the moment that the tank contains 20 gallons of saltwater?

6. Use Euler’s Method with step size $\Delta t = 0.25$ to approximate the solution of the initial value problem

$$\frac{dy}{dt} = (3 - y) (y + 1)$$
$$y(0) = 0.5$$

over the interval $0 \leq t \leq 1$. You must include a table with columns labelled $n$, $t_n$, $y_n$, $f(t_n, y_n)$ (as we have done in class) and you must show at least one sample calculation of these values. (It is sufficient to show how to calculate $t_1$, $y_1$, and $f(t_1, y_1)$.) Do all of your calculations to at least four decimal places.