1. Consider the linear system (written in matrix form)

\[ \frac{dY}{dt} = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix} Y. \]

(a) Check to see whether or not the function

\[ Y_1 = \begin{pmatrix} e^{-3t} \\ e^{-3t} \end{pmatrix} \]

is a solution of the above system. Also, check to see whether or not the function

\[ Y_2 = \begin{pmatrix} e^{-4t} \\ 2e^{-4t} \end{pmatrix} \]

is a solution of the above system.

If you have discovered that either \( Y_1 \) or \( Y_2 \) is not a solution of the above system then stop here. If not, then proceed to part b.

(b) Check to see whether or not the two functions \( Y_1 \) and \( Y_2 \) are linearly independent. If you determine that they are not linearly independent, then stop here. If you determine that they are linearly independent, then proceed to part c.

(c) Find the solution of the above system that satisfies the initial condition

\[ Y(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}. \]

2. Consider the linear system

\[ \frac{dx}{dt} = y, \quad \frac{dy}{dt} = 10x - 3y. \]

(a) Write down the coefficient matrix, \( A \), for this system.

(b) Find the eigenvalues of the matrix, \( A \), and also find an eigenvector associated with each eigenvalue.

(a) Use your results from problem 2 to write down the general solution of the system

\[ \frac{dx}{dt} = y, \quad \frac{dy}{dt} = 10x - 3y. \]
Write the general solution both in vector form:

\[ \mathbf{Y}(t) = \]  

and in standard form:

\[ x(t) = \]  
\[ y(t) = \]

(b) Find the solution of the above system that satisfies the initial condition \( x(0) = -2 \), \( y(0) = 1 \).

3. Consider the brine tank situation described in this diagram.

(a) Letting \( x(t) \) be the amount of salt (in lbs) in Tank X at time \( t \) (in hours) and letting \( y(t) \) be the amount of salt in Tank Y at time \( t \), write down a system of differential equations that models this situation.

\[ \frac{dx}{dt} = \]  
\[ \frac{dy}{dt} = \]

(b) Find the equilibrium solution of this system. Why does this equilibrium solution make intuitive sense?

(a) If you did problem 4 correctly, you should see that the system of differential equations describing the brine tank situation is decoupled and hence easy to solve by methods that you learned earlier in the course. \( \text{(Hint: Remember linear differential equations?)} \) Find the general solution of this system. (Include all details of your solution process.)

(b) Find the particular solution of this system that corresponds to the initial salt concentrations given in problem 4.

(c) Determine the maximum concentration of salt in Tank Y and the time at which this maximum occurs.