1. Each of the functions in a–e is a solution of one of the differential equations in 1–5. Match them.

<table>
<thead>
<tr>
<th>Differential Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.) ( \frac{dy}{dt} = 3y (1 - y) )</td>
<td>a.) ( y = \frac{1}{1 + 5e^{-3t}} )</td>
</tr>
<tr>
<td>2.) ( \frac{dy}{dt} = 3 (1 - y) )</td>
<td>b.) ( y = \frac{3}{1 + 5e^{-3t}} )</td>
</tr>
<tr>
<td>3.) ( \frac{dy}{dt} = y (3 - y) )</td>
<td>c.) ( y = 1 + 5e^{-3t} )</td>
</tr>
<tr>
<td>4.) ( \frac{dy}{dt} = y + e^{3t} )</td>
<td>d.) ( y = \frac{1}{2} (e^{3t} + 5e^{t}) )</td>
</tr>
<tr>
<td>5.) ( \frac{dy}{dt} = y - 3 )</td>
<td>e.) ( y = 3e^{t} + 3 )</td>
</tr>
</tbody>
</table>

Report results here:
1 matches ____________
2 matches ____________
3 matches ____________
4 matches ____________
5 matches ____________

2. Find the general solution of the separable differential equation

\[
\frac{dy}{dt} = t^{2}y^{2}
\]

Then, find the particular solution of this differential equation that satisfies the initial condition \( y(0) = 1/4 \).

Report Results here:
The general solution of the differential equation is ________________
The particular solution satisfying the initial condition \( y(0) = 1/4 \) is __________

3. Solve the linear differential equation

\[
\frac{dy}{dt} = -2y + t.
\]

Then, find the particular solution of this differential equation that satisfies the initial condition \( y(0) = 1/4 \).
The general solution of the differential equation is _________.
The particular solution satisfying the initial condition \( y(0) = 1/4 \) is _________.

4. Consider the linear system of differential equations

\[
\begin{align*}
\frac{dx}{dt} &= 2x - y \\
\frac{dy}{dt} &= -3y.
\end{align*}
\]

(a) The coefficient matrix of this linear system is

\[ A = \]

(b) The eigenvalues of \( A \) are \( \lambda_1 = _____ \) and \( \lambda_2 = _____ \).

(c) An eigenvector for \( \lambda_1 \) is \( v_1 = _____ \) and an eigenvector for \( \lambda_2 \) is \( v_2 = _____ \).

(d) The general solution of the above system is

\[
\begin{align*}
x &= \quad \quad \quad \quad \quad \quad \quad \quad \\
y &= \quad \quad \quad \quad \quad \quad \quad \quad
\end{align*}
\]

(e) The solution of the above system satisfying the initial condition \( x(0) = -2, y(0) = 1 \) is

\[
\begin{align*}
x &= \quad \quad \quad \quad \quad \quad \quad \\
y &= \quad \quad \quad \quad \quad \quad \quad
\end{align*}
\]

5. Since the system in problem 4 is partially decoupled, you should be able to solve it without using eigenvalues and eigenvectors. Do this. (Find the general solution.)

6. Each of the direction fields in a–e matches one of the autonomous systems of differential equations in 1–5. Match them.

1.)

\[
\begin{align*}
\frac{dx}{dt} &= (x + 1)(x - 1) \\
\frac{dy}{dt} &= -y
\end{align*}
\]

2.)

\[
\begin{align*}
\frac{dx}{dt} &= (y + 1)(y - 1) \\
\frac{dy}{dt} &= -x
\end{align*}
\]

\[
2
\]
3.)

\[
\frac{dx}{dt} = (x + 1)(x - 1)
\]
\[
\frac{dy}{dt} = y
\]

4.)

\[
\frac{dx}{dt} = (x + 1)(x - 1)
\]
\[
\frac{dy}{dt} = x
\]

5.)

\[
\frac{dx}{dt} = (x + 1)(y - 1)
\]
\[
\frac{dy}{dt} = -y
\]
Report results here:

1 matches __________
2 matches __________
3 matches __________
4 matches __________
5 matches __________
7. Consider the nonlinear system of differential equations

\[
\begin{align*}
\frac{dx}{dt} &= x(x + y + 3) \\
\frac{dy}{dt} &= -2y(x - y) + 1
\end{align*}
\]

(a) Find the four equilibrium points of this system.
(b) Classify each of the four equilibrium points (as sink, saddle, etc.).
(c) The phase portrait of this system is pictured below. Based on the phase portrait shown and on the information you obtained in part b, describe the behavior of the solution of the above system that satisfies the initial condition

\[
\begin{align*}
x(0) &= -1 \\
y(0) &= -1
\end{align*}
\]

In particular, for this solution, state the values of \( \lim_{t \to \infty} x(t) \), \( \lim_{t \to \infty} y(t) \), \( \lim_{t \to -\infty} x(t) \), and \( \lim_{t \to -\infty} y(t) \).

8. Consider the nonlinear system of differential equations

\[
\begin{align*}
\frac{dx}{dt} &= y(x(1-x^2-y^2)) \\
\frac{dy}{dt} &= x(y(1-x^2-y^2)).
\end{align*}
\]

(a) Show that the equilibrium point \((0, 0)\) is a spiral sink for this system.
(b) Show that

\[
\begin{align*}
x &= \cos t \\
y &= \sin t
\end{align*}
\]

is a solution of this system and draw this solution curve in the phase plane. (Hint: This solution is a periodic solution – meaning that the solution curve in the phase plane is a closed curve.)

(c) Given that the solution curve in part b is the only closed curve solution and also using the fact that \((0, 0)\) is a spiral sink, draw the solution curve of this system that satisfies the initial conditions \(x(0) = 1/2, y(0) = 0\). Also, describe in words what you think happens to this particular solution as \(t \to \infty\) and as \(t \to -\infty\).