Be able to state and apply the **Well-Ordering Principle**, **General Well-Ordering Principle**, and **Principle of Induction**.

Be able to define the following terms:

1. what it means for an integer \( b \neq 0 \) to **divide** an integer \( a \).
2. \( \gcd (a, b) \) and \( \lcm (a, b) \).
3. what it means for two integers, \( a \) and \( b \), to be **relatively prime**.
4. what it means for an integer \( p > 1 \) to be **prime**.
5. the **Sieve of Eratosthenes** (described in a few sentences).

Be able to state and prove the following theorems (propositions, lemmas, etc.):

1. **The Division Algorithm**: Let \( a \) and \( b \) be integers with \( b > 0 \). Then there are unique integers \( q \) and \( r \) such that \( a = qb + r \) and \( 0 \leq r < b \).
2. Suppose that \( a, b, \) and \( c \) are integers such that \( a \) divides \( b \) and \( b \) divides \( c \). Then \( a \) divides \( c \).
3. Suppose that \( a, b, \) and \( c \) are integers such that \( a \) divides \( b \) and \( a \) divides \( c \). Also suppose that \( m \) and \( n \) are any integers. Then \( a \) divides \( mb + nc \).
4. Let \( a \) and \( b \) be integers, not both zero. Then \( \gcd (a, b) \) is the smallest member of the set

\[
S = \{sa + tb \mid s \in \mathbb{Z}, t \in \mathbb{Z}, \text{ and } sa + tb > 0\}.
\]

Furthermore,

\[
S = \{k \cdot \gcd (a, b) \mid k \in \mathbb{N}\}.
\]

5. **Euclid’s Lemma**: Suppose that \( \gcd (a, b) = 1 \) and suppose that \( a \) divides \( bc \). Then \( a \) divides \( c \).
6. If $a, b, q,$ and $r$ are integers (with $a$ and $b$ not both zero) and if $a = qb+r$, then $\gcd(a, b) = \gcd(b, r)$.

7. Let $a$ and $b$ be integers (not both zero). Then every common divisor of $a$ and $b$ divides $\gcd(a, b)$.

8. Let $a$ and $b$ be two non–zero integers. Then $\text{lcm}(a, b)$ divides every common multiple of $a$ and $b$.

9. The **Fundamental Theorem of Arithmetic** (on page 28 of textbook). You just need to be able to state this theorem – not prove it.

10. There is no largest prime number.

   Be able to apply **Euclid’s Algorithm** to find $\gcd(a, b)$.
   Be able to find all (integer) solutions, $(x, y)$, of **Diophantine Equations** of the form $ax + by = c$.
   Be able to solve problems and prove facts of the type encountered in the **homework problems**.