1 What Does The Derivative of $f$ Tell Us About $f$?

**Theorem 1** Suppose that $f$ is a function that is differentiable at all points in some interval $I$.

1. If $f'(x) > 0$ for all $x \in I$, then $f$ is increasing on $I$.

2. If $f'(x) < 0$ for all $x \in I$, then $f$ is decreasing on $I$. 
Example 2 The graph of the derivative, $f'$, of a function $f$ is shown below. What does this tell us about $f$? Suppose that it is also known that $f(0) = 0$. Make a rough sketch of the graph of $f$ in this case.
What Does The Second Derivative of $f$ Tell Us About $f$?

**Theorem 3** Suppose that $f$ is a function that is twice differentiable at all points in some interval $I$.

1. If $f''(x) > 0$ for all $x \in I$, then $f$ is concave up on $I$.

2. If $f''(x) < 0$ for all $x \in I$, then $f$ is concave down on $I$. 
Example 4 Sketch a possible graph of a function, $f$, that satisfies all of the following conditions:

- $f'(x) > 0$ for all $x \in (-\infty, 1)$ and $f'(x) < 0$ for all $x \in (1, \infty)$.
- $f''(x) > 0$ for all $x \in (-\infty, -2)$, and $f''(x) < 0$ for all $x \in (-2, 2)$, and $f''(x) > 0$ for all $x \in (2, \infty)$.
3 Antiderivatives

Definition 5 Suppose that \( f \) is a function whose domain includes some interval \( I \). A function, \( F \), is called an antiderivative of \( f \) on \( I \) if \( F'(x) = f(x) \) for all \( x \in I \).

Example 6 The function \( F(x) = x^2 \) is an antiderivative of the function \( f(x) = 2x \) on the interval \( (-\infty, \infty) \) because (as we saw in an earlier example) \( F'(x) = f(x) \) for all \( x \in (-\infty, \infty) \).

However, the function \( F(x) = x^2 + 6 \) is also an antiderivative of the function \( f(x) = 2x \) on the interval \( (-\infty, \infty) \). In fact, if \( C \) is any constant, then the function \( F(x) = x^2 + C \) is an antiderivative of the function \( f(x) = 2x \) on the interval \( (-\infty, \infty) \). This is why we use the word “an” (rather than “the”) when referring to antiderivatives. When a function \( f \) has an antiderivative on an interval \( I \), then \( f \) always, in fact, has infinitely many antiderivatives on \( I \).

Example 7 Let \( f \) be the function with domain \([0, 5]\) whose graph is shown below and let \( F \) be an antiderivative of \( f \).

![Graph of f](image)

1. On which intervals is \( F \) increasing and on which intervals is \( F \) decreasing?
2. On which intervals is $F$ concave up and on which intervals is $F$ concave down?

3. At which values of $x$ does $F$ have an inflection point?

4. Suppose that $F'(0) = 1$ and make a rough sketch of the graph of $F$.

5. How many antiderivatives does $f$ have?