Instructions. This is a take–home quiz. It is due to be handed in to me by Monday, March 2 at class time. You may work on this quiz alone or in a group of one or two other people. (If you work in a group, then you will hand in only one paper and all in the group will receive the same grade on the quiz.) You may use any books or other resources that you need to do the quiz with the exception of consulting other people. Your solutions must include sufficient detail so that I am able to understand your reasoning process in solving the problems. The paper that you hand in should be written neatly and use correct mathematical notation and writing. Writing, notation and neatness will taken into account in my grading of the quiz. All papers must be stapled together (not paper–clipped or folded). Points will be deducted for no staple.

1. For the function $f(x) = (4x - x^2)(4y - y^2)$,
   a. Complete the following table for $f$.

   \[
   \begin{array}{|c|c|c|}
   \hline
   x & y & f(x,y) \\
   \hline
   0 & 0 & \\
   0 & 2 & \\
   4 & 3 & \\
   1.7 & 4 & \\
   1 & 3 & \\
   3 & 1 & \\
   1 & 1 & \\
   2 & 2 & \\
   3 & 3 & \\
   \hline
   \end{array}
   \]

   b. Use Mathematica to plot a graph of the function $f(x,y) = (4x - x^2)(4y - y^2)$ over the domain $D = [0,4] \times [0,4]$. (Use Plot3D.)

   c. Based on the what you did in parts a and b, at what point in $(x,y) \in D$ does it appear that $f$ achieves it maximum value? What is the maximum value.

   d. Use Mathematica to plot the level curves (contours) of $f$(Use ContourPlot).

2. For the same function $f$, given in Question 1,
   a. Compute the partial derivative, $f_x$, and then compute $f_x(3,1)$.
   b. Compute the partial derivative, $f_y$, and then compute $f_y(3,1)$.
   c. On the graph of $f$, draw small tangent lines at the point $(3,1,9)$ to illustrate the results that you found in parts a and b.
   d. Do the results that you obtained in part a, b, and c makes sense to you in regard to the graph of $f$ at the point $(3,1,9)$? Explain why or why not.

3. For the same function $f$ given in Questions 1 and 2:
a. Compute \( f_{xx} \) and \( f_{yy} \) and then compute \( f_{xx}(3, 1) \) and \( f_{yy}(3, 1) \)

b. Explain what the values of \( f_{xx}(3, 1) \) and \( f_{yy}(3, 1) \) tell you about the graph of \( f \) at the point \((3, 1, 9)\).

c. Compute \( f_{xy} \) and \( f_{yx} \).

d. You should have found in part c that \( f_{xy} = f_{yx} \). Is this unusual? Explain.

4. Suppose that

\[
\begin{align*}
  w &= f(x, y) \\
  x &= g(r, s) \\
  y &= h(r, s) \\
  r &= m(t) \\
  s &= n(t).
\end{align*}
\]

a. Draw a chain diagram that illustrates the connections between the variables \( w, x, y, r, s \) and \( t \) and then write down a Chain Rule for computing \( \frac{\partial w}{\partial t} \).

b. Use the Chain Rule that you wrote down in part a to compute \( \frac{\partial w}{\partial t} \) in the case that

\[
\begin{align*}
  w &= x^2 - 2 \cos(xy) \\
  x &= r^2 - 2s \\
  y &= e^t \\
  r &= t^2 \\
  s &= 3t^3.
\end{align*}
\]

c. For the situation in part b, find \( \left| \frac{\partial w}{\partial t} \right| \big|_{t=1} \). (The answer you should get here is approximately \(-151.22\).)