MATH 2203 (Calculus III) - Quiz 5
April 1, 2015

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Instructions. This is a take–home quiz. It is due to be handed in to me on Wednesday, April 15 at class time. You may work on this quiz alone or in a group of 1–3 other people. (If you work in a group, then you will hand in only one paper and all in the group will receive the same grade on the quiz.) You may use any books or other resources that you need to do the quiz with the exception of consulting other people. Your solutions must include sufficient detail so that I am able to understand your reasoning process in solving the problems. The paper that you hand in should be written neatly and use correct mathematical notation and writing. Writing, notation and neatness will taken into account in my grading of the quiz. All papers must be stapled together (not paper–clipped or folded). Points will be deducted for no staple.

1. Let $C$ be the segment of the parabola $y = x^2 + 2x$ in $\mathbb{R}^2$ with endpoints at $(-3, 3)$ and $(2, 8)$. (A picture of $C$ is given below.) Let

$$f(x, y) = y - x^2 + 2.$$

Evaluate the line integral

$$\int_C f(x, y) \, ds.$$

2. Find the work done by the force field

$$\mathbf{F}(x, y, z) = i - 2ze^{yz}j - 2ye^{yz}k$$

on an object that moves along a straight line path from the point $(-2, 0, -1)$ to the point $(1, 3, 3)$. Your solution must include

a. a parameterization of the path of motion, $C$

b. a line integral that gives the work

c. evaluation of the integral.
3. For the same force field used in problem 2:

\[ \mathbf{F}(x,y,z) = i - 2ze^{yz}j - 2ye^{yz}k, \]

a. Show that \( \mathbf{F} \) satisfies the cross partial conditions.

b. Find a potential function for \( \mathbf{F} \).

c. By appealing to the Fundamental Theorem of Line Integrals, show that the work done by \( \mathbf{F} \) in moving an object along any smooth path, \( C \), from the point \((-2,0,-1)\) to the point \((1,3,3)\) is the same as the answer that you got in problem 2 (assuming that you did problem 2 correctly).