The motion of a particle is given by
\[ x = \frac{3}{2} t^2 \]
\[ y = 2t^2 \]
\[ t > 0. \]

Find

- the position function: \( \mathbf{r}(t) \)
- the velocity: \( \mathbf{v}(t) \) (4 points if you get to here)
- the speed: \( v(t) = |\mathbf{v}(t)| \)
- the acceleration: \( \mathbf{a}(t) \) (10 points if you get to here)
- the unit tangent vector: \( \mathbf{T}(t) \)
- the curvature: \( \kappa(t) \) (16 points if you get to here)
- the tangential component of acceleration: \( a_T(t) \)
- the normal component of acceleration: \( a_N(t) \) (20 points if you get to here)

**Solution:** The vector-valued function for the position of the particle is
\[ \mathbf{r}(t) = \frac{3}{2} t^2 \mathbf{i} - 2t^2 \mathbf{j}. \]

The velocity is
\[ \mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = 3t \mathbf{i} + 4t \mathbf{j} \]
and the speed is

\[ v(t) = |\vec{v}(t)| = \sqrt{(3t)^2 + (4t)^2} = 5t. \]

The acceleration is

\[ \vec{a}(t) = \frac{d\vec{v}}{dt} = 3\hat{i} + 4\hat{j}. \]

The unit tangent vector is

\[ \vec{T}(t) = \frac{1}{v(t)} \vec{v}(t) = \frac{1}{5t} (3\hat{i} + 4\hat{j}) = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}. \]

Also

\[ \frac{d\vec{T}}{dt} = 0 \]

and

\[ \left| \frac{d\vec{T}}{dt} \right| = 0 \]

so the curvature is

\[ \kappa(t) = \frac{1}{v(t)} \left| \frac{d\vec{T}}{dt} \right| = \frac{1}{5t} (0) = 0. \]

The tangential component of acceleration is

\[ a_T(t) = \frac{dv}{dt} = \frac{d}{dt} (5t) = 5 \]

and the normal component of acceleration is

\[ a_N(t) = \kappa(t) (v(t))^2 = (0) (5t)^2 = 0. \]