Instructions. Your work on this exam will be graded according to two criteria: mathematical correctness and clarity of presentation. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using complete sentences where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator but you may not use any books or notes.

1. In $\mathbb{R}^3$, the set of points described by the pair of equations

   \[
   \begin{align*}
   y^2 + z^2 &= 1 \\
   x &= 0
   \end{align*}
   \]

   is (circle the correct choice):

   (a) a circular cylinder of radius 1.
   (b) a circle of radius 1 centered at the point $(0,0,0)$.
   (c) a sphere of radius 1 centered at the point $(0,0,0)$.
   (d) a plane.
   (e) none of the above.

2. Two vectors, $\mathbf{a}$ and $\mathbf{b}$, are pictured below.

   \[
   \mathbf{a} \quad \mathbf{b}
   \]

   (a) Which of the vectors pictured below is $\mathbf{a} + \mathbf{b}$? (Circle the correct choice.)
   
   \[
   \text{W|X|Y|Z}
   \]

   (b) Which of the vectors pictured below is $2\mathbf{a} + \mathbf{b}$? (Circle the correct choice.)
   
   \[
   \text{W|X|Y|Z}
   \]

   (c) Which of the vectors pictured below is $\mathbf{a} - \mathbf{b}$? (Circle the correct choice.)
   
   \[
   \text{W|X|Y|Z}
   \]
(d) Which of the vectors pictured below is $b - a$? (Circle the correct choice.)

\[ \begin{array}{c}
W & X & Y & Z \\
\end{array} \]
3. Let \( u \) and \( v \) be the vectors
\[
    u = i + \sqrt{2}j - \sqrt{2}k \\
    v = -i + j + k
\]
and let \( \theta \) be the angle between \( u \) and \( v \). Compute \( \cos(\theta) \). (You must include all details of your computation in order to get credit.)

**Solution:** We see that
\[
    u \cdot v = (1)(-1) + (\sqrt{2})(1) + (-\sqrt{2})(1) = -1
\]
and
\[
    |u| = \sqrt{1^2 + (\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{5}
\]
and
\[
    |v| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}.
\]
Therefore
\[
    \cos(\theta) = \frac{u \cdot v}{|u||v|} = -\frac{1}{\sqrt{15}} = -\frac{\sqrt{15}}{15}.
\]
(This tells us that \( \theta = \arccos(-\sqrt{15}/15) \approx 104.96^\circ \).)

4. Let \( u \), \( v \), and \( w \) be vectors. Decide whether each of the following expressions is a vector, a scalar, or does not make sense. Circle the correct choices.

(a) \( (u \times v) \times w \) (is a vector / is a scalar / does not make sense).

(b) \( u \times (v \cdot w) \) (is a vector / is a scalar / does not make sense).

(c) \( u \cdot (v \times w) \) (is a vector / is a scalar / does not make sense).

(d) \( (u \times v)w \) (is a vector / is a scalar / does not make sense).

(e) \( (u \cdot v)w \) (is a vector / is a scalar / does not make sense).

5. Find an equation for the plane that contains the points \( P_0 (4, -3, -1) \), \( P_1 (3, 0, 3) \), and \( P_2 (3, -3, 4) \).

**Solution:** To find a normal vector for the plane, we compute
\[
    \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = \begin{vmatrix} i & j & k \\ -1 & 3 & 4 \\ -1 & 0 & 5 \end{vmatrix} = 15i + j + 3k.
\]
Taking \( n \) to be this normal vector, we see that the plane has equation
\[
    n \cdot \overrightarrow{P_0P} = 0
\]
or
\[
    \langle 15, 1, 3 \rangle \cdot \langle x - 4, y + 3, z + 1 \rangle = 0
\]
or
\[
    15(x - 4) + (y + 3) + 3(z + 1) = 0
\]
or
\[
    15x + y + 3z = 54.
\]