1) In answering this question, please use complete sentences and include any pictures that might be helpful in explaining your answers.

a) What is the largest possible subset of $R^2$ that can be used as a domain in defining a function by

$$f(x, y) = \sqrt{y - x}?$$

b) What is the range of $f$?

c) Describe the level curves of this function. Include a picture that shows the general nature of the level curves and also include a written description of the form “The level curves of this function are ______”.

Answers:

a) The largest possible set that can be used as a domain is the set of all points $(x, y) \in R^2$ such that $y \geq x$.

b) The range of $f$ is $[0, \infty)$.

c) The level curve of $f$ corresponding to some given value $c$ in the range of $f$ is

$$\sqrt{y - x} = c.$$ 

This can be written as

$$y - x = c^2$$

or as

$$y = x + c^2.$$ 

Thus we see that the level curves are all lines of slope 1.

2) By considering different paths of approach, show that

$$\lim_{(x,y) \to (0,0)} \frac{x}{\sqrt{x^2 + y^2}}$$

does not exist. Don’t just show calculations. Explain what you are doing.
Solution: Notice that
\[
\lim_{(x,y) \to (0,0)} \frac{x}{\sqrt{x^2 + y^2}} = \lim_{x \to 0^+} \frac{x}{\sqrt{x^2 + 0^2}} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1
\]
and also notice that
\[
\lim_{(x,y) \to (0,0)} \frac{x}{\sqrt{x^2 + y^2}} = \lim_{y \to 0^+} \frac{0}{\sqrt{0^2 + y^2}} = \lim_{y \to 0^+} 0 = 0.
\]
Since we get different values using different paths of approach, we conclude that the indicated limit does not exist.

3) For the function
\[
f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2}},
\]
find \( f_x \) and \( f_y \).

Solution:
First notice that we can write \( f \) as
\[
f(x, y) = (x^2 + y^2)^{-1/2}.
\]
We thus have
\[
f_x = -\frac{1}{2} (x^2 + y^2)^{-3/2} (2x) = \frac{-x}{(x^2 + y^2)^{3/2}}.
\]
Similarly, we obtain
\[
f_y = \frac{-y}{(x^2 + y^2)^{3/2}}.
\]