How Important is Asymmetric Covariance for the Risk Premium of International Assets?

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ABSTRACT
This paper empirically investigates the importance of asymmetric conditional covariance when computing the risk premium of international assets. Conditional second moment asymmetry of equity indices is significant and varies over time. The risk premia estimated allowing for asymmetry are statistically and economically different from risk premia estimated without allowing for asymmetry. In particular, an international investor who ignores covariance asymmetry overestimates required returns for equities of the G4 countries and for the world market, on average.

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Introduction

The international asset pricing models of Solnik (1974), Sercu (1980), and Adler and Dumas (1983) suggest that, when purchase power parity (PPP) does not hold, exposure to foreign exchange risk is priced.\footnote{Adler and Dumas (1983) for instance provide several references showing that Purchase Power Parity does not hold at any time horizon. More recently, Engel (1999), and Perron and Ng (2002) confirm in particular that PPP does not hold even in the long run.} In such a setting, the equilibrium required return of any asset depends on its covariance with the market portfolio, and its covariance with the foreign exchange rates. In other words, it is foreign exchange risk and its pricing that sets models of integrated international capital market aside from domestic models.

The main question this paper asks is, “If investors are compensated for foreign exchange rate risk, in addition to market risk, does it matter whether they take into account conditional covariance asymmetry when estimating the risk premium?” This question is of interest of academics and practitioners alike. For instance, financial institutions and multinational companies use estimates of the risk premium for the evaluation of foreign projects and international M&A activities. Such estimates of the risk premium are important because they affect the discount rate of future cash flows and hence the judgment on which foreign investment opportunities a firm should undertake (see e.g. Bodnar, Dumas, and Marston (2003)).

For individual assets, asymmetric conditional volatility refers to the negative correlation between current returns and future volatility. That is, volatility tends to be lower after a positive return than after a negative return of the same magnitude. Asymmetry in domestic individual stocks and indices is often referred to as the leverage effect. Black (1976) first conjectured that changes in the market value of equity of firms could cause asymmetry. Another hypothesis is that the asymmetric volatility response to returns
shocks could be due to time-varying risk premia. This is often referred to as the *volatility feedback* effect.\(^2\)

A growing body of research investigates variance and covariance asymmetry. Several studies focus mainly on the second moments. In these studies, the risk premium is not the center of attention.\(^3\) Other papers that investigate asymmetry focus on domestic assets. They hence abstract from foreign currency exchange risk.\(^4\) Other works look at asymmetric second moment of assets of foreign countries, but they do so outside the framework of a theoretical model of international capital market equilibrium. They also abstract from foreign currency exchange risk.\(^5\) From these studies, it emerges that asymmetry is a well-established empirical fact in U.S. indices.

The evidence supporting asymmetry is also mounting for international assets. Foreign assets are attractive because they provide potential for diversification beyond domestic assets due to low inter-country correlations. However, Odier and Solnik (1993) and Longin and Solnik (2001) show that these correlations increase in bear market. In other words, the benefits of diversification may not be available when investors need them the most. Longin and Solnik (2001) conclude that the asymmetric correlation pattern should become a key property of any multivariate equity return model to match. It is thus noteworthy that none of the studies mentioned above investigates the importance of asymmetric covariance in the context of an equilibrium model of international capital markets that allows for both market and foreign exchange risk. This paper empirically investigates the importance of asymmetric conditional covariance for the estimation of the risk premium of international assets.

\(^2\)For early work on asymmetry see also Christie (1982), and Brown, Harlow, and Tinic (1988).
\(^3\)See e.g. Kroner and Ng (1998), De Goeij and Marquering (2004), Cappiello, Engle, and Sheppard (2003)
\(^4\)Classic references are Campbell and Hentschel (1992), and Wu (2001).
Dumas and Solnik (1995) show that exchange risk premia for the four largest equity markets, namely U.S., Germany, Japan, and U.K., are non-negligible components of the risk premium. De Santis and Gérard (1998) provide a measure of these foreign exchange risk premia. The existence of foreign exchange risk premia implies that an international investor cannot infer whether covariance asymmetry affects the risk premium of international assets from results based on domestic models. In fact, whether conditional covariance asymmetry matters for asset pricing in international markets is still an outstanding question. This paper aims at filling this gap in the literature.

It is conceivable that the asymmetric response is not only present in international assets, but it also varies over time. Indeed, both the leverage hypothesis and the volatility feedback conjecture are compatible with time variation in the asymmetric response. The leverage effect hypothesis suggests that the asymmetric response is largely due to the change in the firms’ capital structure. If this was the case, then the changes in the relative weights of the market value of equity and debt would determine time variation in the asymmetric response at the firm level. If volatility feedback was the main reason for asymmetry, then investors’ reaction to news, particularly bad news, may be more pronounced depending on the phase of the business cycle. I explicitly test the hypothesis that the asymmetric response may vary over time.

The results of this paper are summarized as follows. In the context of the De Santis and Gérard (1998) conditional implementation of the Adler and Dumas (1983) International Asset Pricing Model (IAPM), I reject the hypothesis of no covariance asymmetry in international assets in favor of the alternative that the covariance responds asymmetrically. In addition, for the equity indices, I find that the asymmetric response significantly varies over time. The main finding is however that the world market risk premium and the equity risk premia estimated without allowing for asymmetry are statistically and economically different from those that obtain allowing for asymmetry. In particular, the
results suggest the an international investor who ignores covariance asymmetry overestimates the required returns from equities. The differences are economically significant, as they range between 62 and 189 basis points per year on average, depending on the international index and the specification of asymmetry considered. For foreign exchange deposits, the risk premia estimated without allowing for asymmetry are significantly different from those that obtain allowing for asymmetry, but they are smaller and also depend on the form of asymmetry considered.

The rest of the paper is organized as follows: Section I presents the empirical implementation of the international asset pricing model. Section II introduces the specification allowing for the constant and time-varying asymmetry in the conditional second moments. Section III presents the estimation methodology and the results. Section IV presents statistical tests highlighting the importance of second moments asymmetry for the estimation of the risk premium. Some economic implications of the results are also discussed in this section. Section V concludes.

I International Asset Pricing Model

This section presents the implementation of the International Asset Pricing Model (IAPM) for the G4 countries: Germany, Japan, UK, and US. Due to the large share of market capitalization relative to the world market, this set of countries is the one that is often the object of study. The IAPM of Adler and Dumas (1983) suggests that exposure to foreign exchange risk should be priced when purchasing power parity (PPP) does not hold. Applied to the G4 markets, the model can be represented as a system of eight equations of the form

\[ E_{t-1}(r_{i,t}) = \delta_{1,t-1} Cov_{t-1}(r_{i,t}, r_{m,t}) + \sum_{c=2}^{4} \delta_{c,t-1} Cov_{t-1}(r_{i,t}, r_{3+c,t}) \]
with $i = 1, 2, 3$ and $4$ for the four equity indices of U.S., Germany, Japan, and U.K., $i = 5, 6,$ and 7 for the 3 foreign currency deposits in DEM, JPY and GBP, and $i = 8$ for the world market portfolio. The returns are expressed in US dollars. The time varying parameter $\delta_{1,t}$, is the price of the world market risk. There are three sources of foreign exchange risk. The time varying parameters $\delta_{2,t}$, $\delta_{3,t}$, and $\delta_{4,t}$ are the prices of the DEM, JPY, and GBP foreign exchange risks, respectively.

De Santis and Gérard (1998) implement this model using a fully parametric approach that allows the simultaneous analysis of international equity market and currency deposits, and the estimation of time-varying conditional prices and measures of risk. They find that the exchange rate risk is priced and provide a measure of its magnitude. Their results extend those in Dumas and Solnik (1995), Harvey (1991), and Ferson and Harvey (1993).

For the purpose of testing, the system of equations above can be conveniently written in matrix form as

$$
\begin{align*}
\mathbf{r}_t &= \delta_{1,t-1}\mathbf{h}_{1,t} + \sum_{c=2}^{4} \delta_{c,t-1}\mathbf{h}_{3+c,t} + \varepsilon_t \\
\varepsilon_t | \mathcal{F}_{t-1} &\sim N(0, \mathbf{H}_t)
\end{align*}
$$

(1)

where $\mathbf{r}_t$ is a $8 \times 1$ vector of returns, $\mathcal{F}_t$ is the set of information available at time $t$. The matrix $\mathbf{H}_t$ is the conditional covariance matrix of the returns, $\mathbf{h}_{1,t}$ is the column of $\mathbf{H}_t$ containing the conditional covariances of each asset with the market, and $\mathbf{h}_{5,t}$, $\mathbf{h}_{6,t}$, $\mathbf{h}_{7,t}$ are the columns of $\mathbf{H}_t$ containing the conditional covariance between each asset and the return on the DEM, JPY and GBP currency deposit respectively. Equation (1) represents a multivariate process consistent with a conditional version of the model of Adler and Dumas (1983). To estimate the model it is necessary to specify the conditional covariance matrix $\mathbf{H}_t$. The results in this paper are based on three specifications for $\mathbf{H}_t$. One benchmark that allows for no asymmetry as in De Santis and Gérard (1998), and
two alternative models that allow for two different types of asymmetric covariance. The symmetric conditional covariance matrix $H_t$ used in the benchmark model is specified as

$$H_t = C' C + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B. \quad (2)$$

For tractability, it is necessary to impose some restriction on the form of the coefficient matrices $A$ and $B$. The diagonal specification for $A$ and $B$ introduced by Engle and Kroner (1995) implies that variances in $H_t$ depend only on past squared residuals and an auto-regressive component. The covariances depend on past cross products and an auto-regressive component. This assumption is restrictive. However, estimation of less restrictive models allowing for second moments spillovers from the world market as in Bekaert and Wu (2000) and Carrieri, Errunza, and Hogan (2005), not presented for brevity, show that spillover effects are generally small for this sample.\(^6\) Therefore, following De Santis and Gérard (1998) I assume that $H_t$ follows a GARCH process with the BEKK diagonality restriction on the coefficient matrices.

All the time-varying prices of risks in (1) are modeled as linear functions of two instruments as in Ferson (1989) and Ferson, Forester and Keim (1993), thus leaving unconstrained the sign of the price of market risk. The instruments are the U.S. default premium ($USDP$) and the U.S. term premium ($USTP$). They are described in detail in the section on data below. The potentially time-varying prices of risk are defined as

$$\delta_{c,t} = \kappa_{c,1} + \kappa_{c,2} USDP_t + \kappa_{c,3} USTP_t, \text{ with } c = 1, \ldots 4. \quad (3)$$

In particular, $\delta_{1,t}$ is the price of the world market risk, $\delta_{2,t}$, $\delta_{3,t}$, and $\delta_{4,t}$ are respectively the world price of foreign exchange risk of the German mark, of the Japanese yen, and

\[^{6}\text{See also de Goeij and Marquering (2004) who argue that spillovers are small.}\]
of the British pound, and $\kappa_{c,1}$, $\kappa_{c,2}$, and $\kappa_{c,3}$ are time invariant parameters.

In the following section, I will use Equation (1) and alternative conditional covariance specifications to investigate the importance of covariance asymmetry for the risk premium of international assets.

II Second Moment Asymmetry

Asymmetric conditional variance denotes the negative correlation between current returns and future variance. The definition of asymmetric conditional covariance is

**Definition 1** Return $r_{i,t}$ and $r_{j,t}$ display asymmetric conditional covariance if for two innovations for asset $i$ $\varepsilon_{i,t}$ and $-\varepsilon_{i,t}$ of a given magnitude but opposite sign and two innovations for asset $j$ $\varepsilon_{j,t}$ and $-\varepsilon_{j,t}$

$$\text{cov}_t[r_{i,t+1}, r_{j,t+1} | \varepsilon_{j,t}, \varepsilon_{i,t}] \neq \text{cov}_t[r_{i,t+1}, r_{j,t+1} | -\varepsilon_{j,t}, -\varepsilon_{i,t}].$$

(4)

I use two alternative parameterizations that can capture covariance asymmetry. The first one is a multivariate variant of the asymmetric specification of Glosten, Jagannathan and Runkle (1993), which makes use of indicator variables.\(^7\) The conditional covariance is specified as

$$H_t = C' C + A' \varepsilon_{t-1} A + B' H_{t-1} B + G' I_{(\varepsilon_{t-1} < 0)} \otimes \varepsilon_{t-1} \varepsilon_{t-1} \otimes I_{(\varepsilon_{t-1} < 0)} G$$

(5)

\(^7\)Bekaert and Harvey (1997) for emerging countries, and Gérard and Wu (2006) use the indicator function to specify asymmetry in a domestic context. Models in Kroner and Ng (1998), and De Goeij and Marquering (2004) also make use indicator functions to capture asymmetry. An alternative approach that deals with asymmetric volatility is in Nelson (1991). The multivariate version of the Nelson’ EGARCH is in Koutmos and Booth (1995) The multivariate version of the GJR model was developed in Hoti, Chan and McAleer (2002), and is sometimes referred to as the VARMA-AGARCH.
where $A$, $B$ are diagonal matrices with typical element $a_{i,i}$ and $b_{i,i}$ and $C$ is triangular, $I_{(e_{t} < 0)}$ is an $8 \times 1$ vector whose elements take value one if the corresponding innovation in vector $e_{t}$ is negative, and $\odot$ is the Hadamard (element by element) product.

The matrix $G$ has the form

$$
\begin{bmatrix}
gamma_{1,1} & \cdots & 0 & \gamma_{1,8} \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \gamma_{7,7} & \gamma_{7,8} \\
0 & \cdots & 0 & \gamma_{8,8}
\end{bmatrix}.
$$

Odier and Solnik (1993), and Longin and Solnik (2001) show that bear markets increase international assets’ dependence. The model in (5) parsimoniously captures asymmetry from negative shocks. The matrix $G$ follows the specification in Bekaert and Wu (2000), and Carrieri, Errunza, and Hogan (2005). This matrix has the desirable property that, in addition to own joint negative shocks, it allows asymmetry to be driven by a negative shock of the world market, and joint negative shocks of the world market and any of the two assets. The parameters $\gamma_{i,8}$ with $i = 1...7$, capture covariance asymmetry spillovers from the world market.

For foreign exchange deposits the appreciation of one currency corresponds to the decline of the other. The notion of “bear market” is thus less clear-cut. It is therefore reasonable to allow for any combination of negative and positive shocks to asymmetrically affect the conditional second moments. In addition, the specification in (5) only allows for time invariant asymmetry. However, both “leverage” and “volatility feedback” are compatible with time variation in the asymmetric response. According to the leverage hypothesis, the asymmetric response is largely due to the change in the firms’ capital structure. If this was the case, then the changes in the relative weights of the market value of equity and debt would determine time variation in the asymmetric response at
the firm level. If volatility feedback was the main reason for asymmetry, then investors’
reaction to news, particularly bad news, may be more or less pronounced depending on
the phase of the business cycle. These considerations provide a motivation for a test
of the empirical alternative hypothesis that the asymmetric response may be driven by
any combination of positive and negative shocks, and that it may vary over time.\(^8\) The
representation of the model with quadratic time varying asymmetry is as follows

\[
H_t = C' + A'(\varepsilon_{t-1} - \theta_{t-1} \odot \sigma_{t-1})(\varepsilon_{t-1} - \theta_{t-1} \odot \sigma_{t-1})'A + B'H_{t-1}B
\]  

(7)

In (7) the 8 x 1 vector of time varying parameters \(\theta_t\) captures the time varying
asymmetry, \(\odot\) is the Hadamard (element by element) product.\(^9\) The 8 x 1 vector \(\sigma_t\)
contains the conditional volatility of \(\varepsilon_t\), i.e. the square root of the diagonal of \(H_t\).
This representation implies that each element of \(\theta_t\) can be interpreted as a standardized
measure of asymmetry at time \(t\). The typical element of the vector \(\theta_t\) is

\[
\theta_{i,t} = \kappa_{i,1}^* + \kappa_{i,2}^*USDP_t + \kappa_{i,3}^*USTP_t, \quad \text{with } i = 1\ldots8.
\]  

(8)

and \(\kappa_{i,1}^*, \kappa_{i,2}^*, \text{ and } \kappa_{i,3}^*\) are time invariant parameters. The instrumental variables
USDP (default premium) and USTP (term premium) can be related to second moment
asymmetry, as the degree of asymmetric response is likely to be influenced by investors’
attitude towards risk. The default premium depends on the expected default loss of
risky bonds and a time-varying risk premium. The term premium may proxy for time-
varying risk aversion. Note that the definition of time varying asymmetry in (8) uses
the same set of instruments and the same functional form as the one used for the price

\(^8\)For a concurrently developed approach to the modeling of time dependent asymmetry see Caporin
and McAleer (2006)

\(^9\)This is type of asymmetry is also known as \textit{N-GARCH}. It appeared first in Engle and Ng (1993).
of risk in equation (3). This ensures that any time variation found in the asymmetry parameter is not the spurious product of an ad hoc choice of instruments or functional forms. One important advantage of these models is that they provide measures of the total risk premium and the risk premia for equities and foreign deposits, which are the objects of the GMM difference test below.

A The Data

The data are weekly observations starting August 1978 and ending December 2004. The sample includes 1379 observations. Returns in U.S. dollars. The equity data are value-weighted total return indexes for the G4 countries, i.e. Germany (DAX 30), Japan (Nikkei 225), U.K. (FTSE 100), and the U.S. (S&P 500). The World Market is the DS-Market total return index. The deposits are 3-month Eurocurrency middle rates for the German mark (DEM), the Japanese yen (JPY), and the British pound (GBP). The excess returns were obtained by subtracting the U.S. 3-month T-Bill rate. The data series used to compute the instruments are the default premium (USDP) defined as the yield difference between the U.S. Moody’s AAA and BAA Corporate Bonds, and the term spread (USTP) defined as the difference between the U.S. 10 year Treasury with constant maturity and the U.S. 3-month T-Bill. The instruments are then first order differenced to treat the non-stationarity of the levels, and demeaned to facilitate the interpretation of the results. All data are from Datastream.\(^\text{10}\) The default premium (USDP) and the term premium (USTP) have been widely used in international asset pricing literature, and are now considered to some extent as standard instrumental variables. Avramov (2002) finds that the term premium is robust both in sample and out of sample. He also notices that the USTP’s good performance is due to its ability to capture the exposure related to shifts in interest rates and economic conditions that affect the likelihood of

\(^{10}\)These data are all well known. Descriptive statistics are omitted to conserve space.
default. In particular, it is possible that the degree of asymmetric response could be influenced by investors’ attitudes towards risk. Since the term premium may proxy for time-varying risk aversion, $USTP$ is a natural candidate variable for explaining the variation over time of second moment asymmetry. The default spread ($USDp$), is also intuitively appealing to explain asymmetry as it could capture some aggregate measure of firms' leverage.\textsuperscript{11}

### III Estimation

This section presents estimation and statistical testing results, Wald tests for joint hypotheses, and LR tests for model specification. More insights into the implications of asymmetry for the estimation of the risk premia and specific tests appear in the subsequent section.

All the models estimated belong to the multivariate $GARCH(1,1)$ in mean family. Under the assumptions described in the previous sections, the loglikelihood function can be written as

$$\ln L(\psi) = -\frac{T k}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \ln |H_t(\psi)| - \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t(\psi)'H_t(\psi)^{-1}\varepsilon_t(\psi)$$

where $\psi$ is the vector of all unknown parameters, $T$ is the number of observations, and $k$ is the number of assets.

To account for the fact that in financial time series the normality assumption is often violated, all the models were estimated using the Quasi-Maximum Likelihood (QML) approach proposed by Bollerslev and Wooldridge (1992). Under mild regularity

\textsuperscript{11}The World Market Dividend Yield was found to have little explanatory power and it was not used. In addition, the robustness of dividend yield as return predictor has been recently put into question. Goyal and Welch (2003) find that the dividend yield forecasts returns only over horizons longer than 5-10 years.
conditions, the QML estimator is consistent and asymptotically normal. The robust variance-covariance matrix is computed as $D^{-1}SD^{-1}T^{-1}$, where $S$ is the expected value of the cross product of the scores and $D$ is the negative of the expected value of the Hessian. $S$ and $D$ were computed using two sided numerical gradient and Hessian respectively.\textsuperscript{12}

\section{Empirical Results}

Tables I, II, and III show the parameters estimates of the benchmark and the two alternative models. The robust t-stats are in parentheses. GARCH and ARCH parameters are highly significant in almost all cases with magnitudes comparable to those reported in previous studies and will not be discussed.

For the purpose of estimation, the information variables USDP and USTP were demeaned. This implies that the estimate of the constant coefficients $\kappa_{c,1}$ in equation (3) can be interpreted as measures of the unconditional price of risks $c$ for each source of risk. The benchmark model has no asymmetry. Parameters estimates are shown in Table I.

\begin{center}
[Insert table I about here.]
\end{center}

The constant parameter of the price of world market risk is 0.035, but with a t-stat of 1.311. The results of the Wald tests for the benchmark model are reported in Panel 2. The first test shows that risk is priced in international markets. The second test shows that also currency risk is priced. The third test shows that the world market risk is unconditionally priced, as there is no evidence that the price of world market risk varies

\textsuperscript{12}The likelihood function is maximized using a multiple-start generalized hill-climbing algorithm robust to local optima. The tolerance, defined as the minimum amount of absolute improvement in the loglikelihood function value required each iteration, was set to $10^{-8}$. The results were also verified using BFGS Quasi-Newton method using an array of starting values.
over time. Most importantly, the fourth Wald test confirms that foreign exchange risk is conditionally priced, with a p-stat of 0.013. These results closely mirror those in De Santis and Gérard (1998). Their estimate of the unconditional price of market risk using monthly data is 0.0279, and they were first to show that exchange risk is conditionally priced.

The parameters estimates of the GJR model are shown in Table II. The average price of market risk estimate is 0.031. The first Wald test in Panel 2 shows that market and currency risk are jointly priced. The second Wald test shows that the GJR asymmetry parameters $\gamma_{i,t}$, for all $i$’s are jointly significant at any conventional level. In the third and fourth tests, asymmetry is also statistically significant for both equities and foreign deposits separately. The asymmetric covariance spillovers from world market’s negative shocks, and simultaneous negative shocks to the market and any of the two assets are captured by the parameters $\gamma_{i,t}$, with $i = 1...7$. The fifth Wald test shows that market spillover parameters are jointly significant at any conventional level. This result gives support to the non-diagonal specification of the $G$ matrix shown in (6). In summary, the Wald tests in Panel 2 provide strong evidence in support of the hypothesis that asymmetry from joint negative shocks, as captured by the non diagonal GJR specification, is statistically significant.

[Insert table II about here.]

Table III shows the parameters estimates of the model using time-varying quadratic asymmetry $\theta_{i,t}$. The average price of market risk estimate is 0.027, significant at any conventional level. Panel 2 presents Wald tests of joint statistical significance of the asymmetry parameters. The first test rejects the hypothesis that international returns display symmetric covariance. The second rejects the hypothesis that asymmetry is constant with a p-value of 0.061. In addition, the third and fourth Wald tests show that
equities display time-varying asymmetric covariance. This result is novel and is one of the contributions of this paper. The fifth test shows that the quadratic model does not detect asymmetry for foreign deposits.

[Insert table III about here.]

The likelihood ratio tests, not presented for brevity, strongly reject the benchmark when compared with either alternative model (p-val < 0.001 for both tests).

IV The Estimated Risk Premium

The previous section establishes the statistical significance of asymmetry for international assets of the G4 countries. Equation (1) shows that the required return of any asset depends on its covariance with the market portfolio, and its covariance with the foreign exchange rates. A question that this paper addresses is whether asymmetry in the conditional second moments does affect the estimates of the risk premia. The results of the tests below show that risk premia estimated with the symmetric benchmark are statistically and economically different from the risk premia estimated with the two alternative models in turn. More precisely, the risk premia differences are not only statistically significant, but their average size is such that it can affect investment decisions and outcomes, and hence is economically important. In the following section, the risk premia estimated from the three models are formally compared using a GMM test of the risk premium differences.

A Economic Importance of the Risk Premia Differences

Financial institutions and multinational companies use estimates of the risk premium for the evaluation of foreign projects and international M&A activities (see e.g.
Bodnar, Dumas, and Marston (2003)). Such estimates of the risk premium are important because they affect the discount rate of future cash flows and hence the judgment on which foreign investment opportunities a firm should undertake. Active portfolio managers can increase portfolio weights of assets with superior risk premia. It follows that for decision makers large, either positive or negative, differences between risk premia estimated using competing models have important economic implications. To assess the economic importance of asymmetry for the estimation of the risk premium it is thus necessary to determine if the risk premia from asymmetric models are sizably and significantly different from those from the symmetric benchmark.

The GMM framework provides tools to measure the size and significance of the difference between the risk premia estimated with competing models. The tests consist in GMM regressing the time series of the differences of estimated risk premia on a constant. The only instrument used to define the information set is a constant.

Regardless of the covariance specification, let the total estimated risk premium \( RP_{i,t} \) at time \( t \) for asset \( i \), with \( i = 1...8 \) be defined as

\[
RP_{i,t} = \delta_{1,t-1} Cov_{t-1}(r_{i,t}, r_{m,t}) + \sum_{c=2}^{4} \delta_{c,t-1} Cov_{t-1}(r_{i,t}, r_{3+c,t}).
\]  

Each model in the previous section yields an estimate of the risk premium \( RP_{i,t} \) for each of the 8 assets. These estimates differ from one another depending on the conditional covariance specification. For instance, for each asset \( i \), the test of the difference in the risk premium estimated from the benchmark and from the GJR is performed by running the following GMM regression

\[
RP_{i,t}^{(GJR)} - RP_{i,t}^{(B)} = \phi_i + \epsilon_{i,t}
\]

where \( RP_{i,t}^{(GJR)} \) and \( RP_{i,t}^{(B)} \) are the risk premium estimates from the GJR model and the
benchmark model respectively, $\phi_i$ is a constant, and $\epsilon_{i,t}$ is a potentially autocorrelated and heteroskedastic residual. In equation (11), the value of $\phi_i$ is the sample mean of the risk premia differences, which is insensitive to the choice of instruments. Under the null hypothesis that the risk premia $RP_{i,t}^{(GJR)}$ and $RP_{i,t}^{(B)}$ are the same, $\phi_i$ should be zero. The difference between the risk premium from the time varying asymmetric model $RP_{i,t}^{(TVA)}$ and $RP_{i,t}^{(GJR)}$ is tested in a similar way. The significance of the $\phi_i$'s is tested using standard errors corrected using the Newey and West (1987) estimator with the Andrews (1991) bandwidth selection, robust to autocorrelation and heteroskedasticity.

Table IV shows the results for the GMM test of the risk premium difference across the three models. The t-stats show that models with different covariance specifications yield estimates of the risk premium that are significantly different from one another on average.\textsuperscript{13} An interesting regularity emerges from Table IV. The inclusion of asymmetry yield lower risk premia estimates for equities. Figure 1 shows this pattern. The risk premia estimates difference range between 62 and 189 basis points per year, on average, depending on the international asset and the specification of asymmetry considered. In the case of US stock market for instance, difference between risk premia estimates is 189 basis points when using quadratic asymmetry and 92 basis points when using the GJR model per year.\textsuperscript{14} For foreign exchange deposits the differences are significant on average, but smaller and they also depend on the form of asymmetry considered. It is not surprising that the test results for the two asymmetry specifications do not coincide as the GJR specification captures only the effects of joint negative shocks.

\textsuperscript{13}Alternative specifications of the GMM regressions using lagged premia and lagged differences as instruments are similarly significant at any conventional level.

\textsuperscript{14}The results of the GMM test differences are robust to the more flexible specification of the matrices A and B in all models as in Bekaert and Wu (2000), Carrieri, Errunza, and Hogan (2005). However, this more general covariance specification, and the additional parameters required, reduce the precision of the risk premium estimate. Results are not presented for brevity, but are available upon request.
These discrepancies are also not surprising for the currency deposits as for currencies the notion of decline is not as clear-cut as it is for equities.

[Insert table IV and Figure 1 about here.]

To further appreciate the economic importance of the difference in the estimated risk premia, consider for instance an investor from U.S. who has an investment opportunity located in Japan with risk characteristics similar to those of the MSCI index. She can use the estimated risk premium for the Japanese equity as a measure of the equilibrium required return for the equity component of the project. As shown in table IV however the average difference between the risk premium computed with the symmetric model and the risk premium computed with the asymmetric models can be between negative 88 (GJR model) and 157 basis points (Time-varying asymmetric) per year. It follows that if she relies on the symmetric model to compute the required return, she will wrongly consider unacceptable too many projects and systematically discard them. Likewise, an international portfolio manager engaged in tactical allocation can use the estimated risk premia to tilt her global portfolio in the direction of country assets with superior risk premia. The results in Table IV suggest that not allowing for asymmetry in the estimation of the risk premia leads to a sizable over statement of expected performance on average.

The economic theory developed in Adler and Dumas (1983) signifies that the estimate of the world price of market risk $\delta_{1,t}$ is a measure investors’ risk aversion. In the terms of the theoretical model, $\delta_{1,t}$ is the inverse of the weighted average risk tolerance of the four country’s investors, where the weights are the wealth shares of each country. The risk tolerance, which measures the curvature of the indirect utility function, is also dependent on wealth. Wealth of international investors is difficult to measure as, in general, investors hold foreign assets and hence markets capitalizations are of little help. However, as the value of these assets fluctuates and investors adjust their portfolios,
so does investors’ wealth. It is therefore consistent with the theoretical model that the price of world market risk is allowed to vary. It is thus reassuring that $\delta_1,t$ is positive on average regardless of the parameterization, as the price of market risk can be negative only if the risk aversion is negative. Figure 2 shows that the prices of market risk estimated from competing models display comparable dynamics. The quadratic time-varying asymmetry parameterization appears to have a distinct effect on the level of the estimated prices of risks. In particular, the price of market risk estimated with quadratic asymmetry is lower than that estimated with either the benchmark or the model with GJR asymmetry. In terms of the Adler and Dumas (1983) theoretical set up, this result implies that the equilibrium model allowing for quadratic asymmetry is consistent with investors who are less risk averse than the symmetric model suggests. Figure 2 also suggests that the risk premium differences shown in Table IV are driven largely by the conditional covariance estimates in the case of the GJR model. For the quadratic model, the estimated risk aversion appear to have a larger impact on the risk premia estimates.

Historically, the evolution over time of the risk premia seems related to events preceding the introduction of the Euro currency. Especially for the GBP, the peaks around 1992 and 1993 coincide with the European Monetary System crises. The uncertainty started to dissipate in January 1994 with the start of the second stage of EMU. In December 1995, the European Council agreed to name the European currency unit to be introduced at the start of Stage Three, the “euro”, and confirmed that Stage Three of EMU would start on January 1, 1999.

V Conclusion

This paper investigates the importance of covariance asymmetry with respect to the estimation of the risk premium of international assets. The overall result is that
risk premia of the G4 countries’ assets estimated as in De Santis and Gérard (1998) are statistically and economically different form risk premia estimated with otherwise identical models augmented to account for covariance asymmetry.

The main findings are as follows. Covariance asymmetry is not only present, but is also significantly time-varying for the equities of developed countries. In addition, the world market and foreign exchange risk premia estimated allowing for asymmetry are statistically and economically different from those computed without allowing for asymmetry. Tests for the differences in the risk premium estimated with or without asymmetry suggest that an international investor who overlooks covariance asymmetry over estimates the risk premium from equities. In particular, an international investor who ignores covariance asymmetry overestimates required returns from equities between 62 and 189 basis points per year on average depending on the asset and on the alternative asymmetric model. For foreign exchange deposits the risk premia differences are significant, but smaller and also depend on the form of asymmetry considered.
References


The benchmark model is a multivariate symmetric M-GARCH(1,1). The specification is 
\[ r_{i,t} = \delta_{1,t-1} \text{Cov}_{t-1}(r_{i,t}, r_{m,t}) + \sum_{c=2}^{4} \delta_{c,t-1} \text{Cov}_{t-1}(r_{i,t}, r_{c+3,t}) + \varepsilon_{i,t} \text{ with } i = 1, \ldots, 8. \]

The error term is \( \varepsilon_{t} \sim N(0, H_{t}) \). The potentially time-varying prices of risks are defined as \( \delta_{c,t} = \kappa_{c,1} + \kappa_{c,2} \text{USDP}_{t} + \kappa_{c,3} \text{USTP}_{t}, c = 1, \ldots, 4 \). In particular, \( \delta_{1,t-1} \) is the world price of market risk, \( \delta_{2,t}, \delta_{3,t}, \) and \( \delta_{4,t} \) are respectively the world price of foreign exchange risk of the German mark, of the Japanese yen, and of the British pound. The parameters \( \kappa_{c,1}, \kappa_{c,2}, \) and \( \kappa_{c,3} \) are time invariant. All returns are measured in USD, there are three sources of foreign exchange risk: DEM, JPY, and GBP. The instrumental variables are the default premium (USDP), and the term premium (USTP). The conditional covariance is specified as 
\[ H_{t} = C' C + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B, \]
where \( A \) and \( B \) are diagonal matrices with typical element \( \alpha_{i} \) and \( \beta_{i} \) and \( C \) is triangular. Panel 1 presents the parameters estimates of the benchmark model. The robust t-stats are in parentheses. T-stats larger than two are in bold. Panels 2 shows Wald tests of significance and variability of the prices of risks from the benchmark model. The tests show that, market risk, and foreign exchange risks are priced. Also, the foreign exchange prices of risks vary over time. The null that the price of market risk is constant is not rejected.

<table>
<thead>
<tr>
<th>( \kappa_{c,1} )</th>
<th>( \delta_{1,t} )</th>
<th>( \delta_{2,t} )</th>
<th>( \delta_{3,t} )</th>
<th>( \delta_{4,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_{c,1} )</td>
<td>0.035 (1.311)</td>
<td>0.001 (-0.341)</td>
<td>-0.005 (0.036)</td>
<td>-0.001 (-0.067)</td>
</tr>
<tr>
<td>( \kappa_{c,2} )</td>
<td>-0.216 (-0.661)</td>
<td>-0.230 (1.090)</td>
<td>0.458 (0.036)</td>
<td>0.517 (-0.341)</td>
</tr>
<tr>
<td>( \kappa_{c,3} )</td>
<td>-0.037 (-0.680)</td>
<td>0.175 (1.694)</td>
<td>-0.027 (-0.036)</td>
<td>0.016 (-0.341)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \alpha_{1,1} )</th>
<th>( \alpha_{2,2} )</th>
<th>( \alpha_{3,3} )</th>
<th>( \alpha_{4,4} )</th>
<th>( \alpha_{5,5} )</th>
<th>( \alpha_{6,6} )</th>
<th>( \alpha_{7,7} )</th>
<th>( \alpha_{8,8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{1,1} )</td>
<td>0.246 (13.309)</td>
<td>0.232 (17.981)</td>
<td>0.210 (6.958)</td>
<td>0.218 (16.503)</td>
<td>-0.009 (-0.265)</td>
<td>-0.093 (-1.832)</td>
<td>0.199 (8.543)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \beta_{1,1} )</th>
<th>( \beta_{2,2} )</th>
<th>( \beta_{3,3} )</th>
<th>( \beta_{4,4} )</th>
<th>( \beta_{5,5} )</th>
<th>( \beta_{6,6} )</th>
<th>( \beta_{7,7} )</th>
<th>( \beta_{8,8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{1,1} )</td>
<td>0.965 (164.527)</td>
<td>0.971 (302.292)</td>
<td>0.974 (94.639)</td>
<td>0.975 (460.521)</td>
<td>0.987 (67.644)</td>
<td>0.975 (87.494)</td>
<td>0.967 (112.018)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \chi^{2} )</th>
<th>( \text{p-val} )</th>
<th>( \text{df} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^{2} )</td>
<td>38.45 (164.527)</td>
<td>0.000 (0.965)</td>
</tr>
</tbody>
</table>

**Panel 1**

**Panel 2**
Table II
The alternative model with GJR asymmetry is specified as
\[ r_{i,t} = \delta_1 \text{Cov}_t - 1 (r_{i,t}; r_{m,t}) + \sum_{s=2}^{4} \delta_{s,t-1} \text{Cov}_{t-1} (r_{i,t}; r_{s+t}) + \varepsilon_{i,t}, \]
with, \( i = 1, \ldots, 8 \). The error term is \( \varepsilon_{i,t} \sim N(0, H_t) \). The potentially
time-varying prices of risks are defined as \( \delta_{s,t} = \kappa_{c,1} + \kappa_{c,2} \text{USD}_{P_t} + \kappa_{c,3} \text{UST}_{P_t}, \ c = 1, \ldots, 4 \). In particular,
\( \delta_{1,t-1} \) is the world price of market risk, \( \delta_{2,t} \), \( \delta_{3,t} \), and \( \delta_{4,t} \) are respectively the world price of foreign exchange
risk of the German mark, of the Japanese yen, and of the British pound. The parameters \( \kappa_{c,1}, \kappa_{c,2}, \) and \( \kappa_{c,3} \) are time invariant. The instruments are the default premium
of USD\( P_t \), and the term premium of USD\( T_{P_t} \). The covariance specification is \( H_t = C' C + A' \varepsilon_{t-1} \varepsilon'_{t-1} A + B' H_{t-1} B + G'I_{(\varepsilon_{t-1} < 0)} \varepsilon_{t-1} \varepsilon'_{t-1} \cap I_{(\varepsilon_{t-1} < 0)} G \),
where \( A, B, C \) are diagonal with typical element \( \alpha_{i,i} \) and \( \beta_{i,i} \), and \( C \) is triangular. The matrix \( G \) has a non zero
main diagonal and last column as in Bekaert and Wu (2000). \( I_{(\varepsilon_{t-1} < 0)} \) is an 8 \( \times \) 1 vector whose elements take
value one if the corresponding innovation in vector \( \varepsilon_t \) is negative, and \( \cap \) is the Hadamard product. Panel 1 shows parameters estimates of the GJR asymmetric model. The robust t-stats are in parentheses. T-stats
larger than two are in bold. Panel 2 show Wald tests of significance and variability of the prices of risks, and
significance of the constant asymmetry from the GJR model.

Panel 1

<table>
<thead>
<tr>
<th>( \kappa_{c,1} )</th>
<th>( \delta_{1,t} )</th>
<th>( \delta_{2,t} )</th>
<th>( \delta_{3,t} )</th>
<th>( \delta_{4,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.031</td>
<td>0.002</td>
<td>-0.007</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>(1.431)</td>
<td>(0.069)</td>
<td>(-0.387)</td>
<td>(-0.027)</td>
<td></td>
</tr>
<tr>
<td>( \kappa_{c,2} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.287</td>
<td>-0.260</td>
<td>0.541</td>
<td>0.568</td>
<td></td>
</tr>
<tr>
<td>(-1.514)</td>
<td>(-0.506)</td>
<td>(1.285)</td>
<td>(1.748)</td>
<td></td>
</tr>
<tr>
<td>( \kappa_{c,3} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.038</td>
<td>0.156</td>
<td>-0.021</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>(-0.405)</td>
<td>(1.074)</td>
<td>(-0.229)</td>
<td>(0.300)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \gamma_{1,1} )</th>
<th>( \gamma_{1,2} )</th>
<th>( \gamma_{1,3} )</th>
<th>( \gamma_{1,4} )</th>
<th>( \gamma_{1,5} )</th>
<th>( \gamma_{1,6} )</th>
<th>( \gamma_{1,7} )</th>
<th>( \gamma_{1,8} )</th>
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<tbody>
<tr>
<td>-0.037</td>
<td>-0.079</td>
<td>-0.018</td>
<td>-0.067</td>
<td>-0.173</td>
<td>0.217</td>
<td>0.121</td>
<td>0.117</td>
</tr>
<tr>
<td>(-0.129)</td>
<td>(-0.915)</td>
<td>(-0.016)</td>
<td>(-0.889)</td>
<td>(-0.124)</td>
<td>(2.063)</td>
<td>(0.247)</td>
<td>(1.177)</td>
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</table>

<table>
<thead>
<tr>
<th>( \gamma_{1,8} )</th>
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<th>( \gamma_{4,8} )</th>
<th>( \gamma_{5,8} )</th>
<th>( \gamma_{6,8} )</th>
<th>( \gamma_{7,8} )</th>
<th>( \gamma_{8,8} )</th>
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</thead>
<tbody>
<tr>
<td>0.269</td>
<td>0.193</td>
<td>0.157</td>
<td>0.168</td>
<td>-0.010</td>
<td>-0.066</td>
<td>-0.019</td>
<td></td>
</tr>
<tr>
<td>(1.395)</td>
<td>(0.619)</td>
<td>(0.165)</td>
<td>(1.111)</td>
<td>(-0.010)</td>
<td>(-0.286)</td>
<td>(-0.161)</td>
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<table>
<thead>
<tr>
<th>( \alpha_{1,1} )</th>
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<th>( \alpha_{4,4} )</th>
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<th>( \alpha_{6,6} )</th>
<th>( \alpha_{7,7} )</th>
<th>( \alpha_{8,8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.229</td>
<td>0.224</td>
<td>0.204</td>
<td>0.216</td>
<td>0.118</td>
<td>-0.126</td>
<td>0.181</td>
<td>0.221</td>
</tr>
<tr>
<td>(14.384)</td>
<td>(6.066)</td>
<td>(1.333)</td>
<td>(5.452)</td>
<td>(0.253)</td>
<td>(-1.198)</td>
<td>(2.476)</td>
<td>(9.345)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \beta_{1,1} )</th>
<th>( \beta_{2,2} )</th>
<th>( \beta_{3,3} )</th>
<th>( \beta_{4,4} )</th>
<th>( \beta_{5,5} )</th>
<th>( \beta_{6,6} )</th>
<th>( \beta_{7,7} )</th>
<th>( \beta_{8,8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.965</td>
<td>0.971</td>
<td>0.974</td>
<td>0.973</td>
<td>0.672</td>
<td>0.936</td>
<td>0.968</td>
<td>0.971</td>
</tr>
<tr>
<td>(192.807)</td>
<td>(221.720)</td>
<td>(24.521)</td>
<td>(231.293)</td>
<td>(0.206)</td>
<td>(7.393)</td>
<td>(64.950)</td>
<td>(359.721)</td>
</tr>
</tbody>
</table>

Panel 2

<table>
<thead>
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<th>( \chi^2 )</th>
<th>p-val</th>
<th>df</th>
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</thead>
<tbody>
<tr>
<td>20.72</td>
<td>0.055</td>
<td>12</td>
</tr>
<tr>
<td>99.61</td>
<td>0.000</td>
<td>8</td>
</tr>
<tr>
<td>64.88</td>
<td>0.000</td>
<td>5</td>
</tr>
<tr>
<td>15.81</td>
<td>0.001</td>
<td>3</td>
</tr>
<tr>
<td>58.83</td>
<td>0.000</td>
<td>7</td>
</tr>
</tbody>
</table>
The time varying asymmetry parameter is the British pound. The parameters respectively the world price of foreign exchange risk of the German mark, of the Japanese yen, and of the potentially time-varying prices of risks are defined as $\delta_{c,t} = \kappa_{c,1} + \kappa_{c,2}USDP_t + \kappa_{c,3}USTP_t$, $c = 1,...4$. In particular, $\delta_{1,t-1}$ is the world price of market risk, $\delta_{2,t}$, $\delta_{3,t}$, and $\delta_{4,t}$ are respectively the world price of foreign exchange risk of the German mark, of the Japanese yen, and of the British pound. The parameters $\kappa_{c,1}$, $\kappa_{c,2}$, and $\kappa_{c,3}$ are time invariant. The instrumental variables are the default premium (USDP), and the term premium (USTP). The conditional covariance is $H_t = CC' + A(\varepsilon_{t-1} - \theta_{t-1} \odot \sigma_{t-1})' (\varepsilon_{t-1} - \theta_{t-1} \odot \sigma_{t-1}) A' + BH_{t-1} B'$, where A and B are diagonal. The time varying asymmetry parameter is $\theta_{i,t}$, with $i = 1...8$, $\odot$ denotes the Hadamard (element by element) product, and $\sigma_t$ is the square root of the main diagonal of $H_t$. The typical element of $\theta_{i,t} = \kappa_{i,1} + \kappa_{i,2}USDP_t + \kappa_{i,3}USTP_t$, with $i = 1...8$. The parameters $\kappa_{i,1}$, $\kappa_{i,2}$, and $\kappa_{i,3}$ are time invariant. Panel 1 shows the parameters estimates of the time-varying quadratic asymmetric model. The robust t-stats are in parentheses. T-stats larger than two are in bold. Panel 2 shows Wald tests of significance and variability of covariance asymmetry.

<table>
<thead>
<tr>
<th>Panel 1</th>
<th>$\delta_{1,t}$</th>
<th>$\delta_{2,t}$</th>
<th>$\delta_{3,t}$</th>
<th>$\delta_{4,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_{c,1}$</td>
<td>0.027</td>
<td>0.005</td>
<td>-0.001</td>
<td>0.009</td>
</tr>
<tr>
<td>(2.082)</td>
<td>(0.251)</td>
<td>(-0.048)</td>
<td>(0.477)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{c,2}$</td>
<td>-0.041</td>
<td>-0.215</td>
<td>0.459</td>
<td>0.442</td>
</tr>
<tr>
<td>(-0.185)</td>
<td>(-0.728)</td>
<td>(1.452)</td>
<td>(1.483)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{c,3}$</td>
<td>-0.049</td>
<td>0.170</td>
<td>-0.041</td>
<td>0.020</td>
</tr>
<tr>
<td>(-0.651)</td>
<td>(1.841)</td>
<td>(-0.513)</td>
<td>(0.234)</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.030</td>
<td>0.129</td>
<td>0.098</td>
<td>0.102</td>
</tr>
<tr>
<td>(0.468)</td>
<td>(2.883)</td>
<td>(1.163)</td>
<td>(1.770)</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.178</td>
<td>-2.244</td>
<td>-1.335</td>
<td>-0.763</td>
</tr>
<tr>
<td>(-0.103)</td>
<td>(-2.638)</td>
<td>(-1.095)</td>
<td>(-0.014)</td>
<td></td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>0.127</td>
<td>0.003</td>
<td>0.323</td>
<td>-0.141</td>
</tr>
<tr>
<td>(0.439)</td>
<td>(0.013)</td>
<td>(0.866)</td>
<td>(-0.557)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{1,1}$</td>
<td>0.224</td>
<td>0.219</td>
<td>0.208</td>
<td>0.207</td>
</tr>
<tr>
<td>(11.511)</td>
<td>(17.199)</td>
<td>(12.002)</td>
<td>(16.270)</td>
<td></td>
</tr>
</tbody>
</table>

Panel 2

Do international returns display symmetric covariance?
$H_0$: The coefficients $\theta_i = 0$ for all $i$'s

Do international returns display constant asymmetric covariance?
$H_0$: The coefficients $\kappa_{i,2} = \kappa_{i,3} = 0$ for $i = 1,..., 8$

Do international equities display symmetric covariance?
$H_0$: The coefficients $\theta_i = 0$ for $i = 1,...,4$, and 8

Do international equities display constant symmetric covariance?
$H_0$: The coefficients $\kappa_{i,2} = \kappa_{i,3} = 0$ for $i = 1, 2, 3, 4$ and 8

Do foreign deposits display symmetric covariance?
$H_0$: The coefficients $\theta_i = 0$ for $i = 5, 6$, and 8

<table>
<thead>
<tr>
<th>$\chi^2$</th>
<th>p-val</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>94.93</td>
<td>0.000</td>
<td>24</td>
</tr>
<tr>
<td>25.54</td>
<td>0.061</td>
<td>16</td>
</tr>
<tr>
<td>75.43</td>
<td>0.000</td>
<td>15</td>
</tr>
<tr>
<td>19.05</td>
<td>0.040</td>
<td>10</td>
</tr>
</tbody>
</table>

Table III

The alternative model with time varying asymmetric covariance is $r_{i,t} = \delta_{1,t-1} Cov_{t-1}(r_{i,t}, r_{m,t}) + \sum_{c=2}^{4} \delta_{c,t-1} Cov_{t-1}(r_{i,t}, r_{c,t}) + \varepsilon_{i,t}$ with $i = 1,...,8$. The error term is $\varepsilon_{i,t} \sim N(0, H_t)$. The potentially time-varying prices of risks are defined as $\delta_{c,t} = \kappa_{c,1} + \kappa_{c,2}USDP_t + \kappa_{c,3}USTP_t$, $c = 1,...4$. In particular, $\delta_{1,t-1}$ is the world price of market risk, $\delta_{2,t}$, $\delta_{3,t}$, and $\delta_{4,t}$ are respectively the world price of foreign exchange risk of the German mark, of the Japanese yen, and of the British pound. The parameters $\kappa_{c,1}$, $\kappa_{c,2}$, and $\kappa_{c,3}$ are time invariant. The instrumental variables are the default premium (USDP), and the term premium (USTP). The conditional covariance is $H_t = CC' + A(\varepsilon_{t-1} - \theta_{t-1} \odot \sigma_{t-1})' (\varepsilon_{t-1} - \theta_{t-1} \odot \sigma_{t-1}) A' + BH_{t-1} B'$, where A and B are diagonal. The time varying asymmetry parameter is $\theta_{i,t}$, with $i = 1...8$, $\odot$ denotes the Hadamard (element by element) product, and $\sigma_t$ is the square root of the main diagonal of $H_t$. The typical element of $\theta_{i,t} = \kappa_{i,1} + \kappa_{i,2}USDP_t + \kappa_{i,3}USTP_t$, with $i = 1...8$. The parameters $\kappa_{i,1}$, $\kappa_{i,2}$, and $\kappa_{i,3}$ are time invariant. Panel 1 shows the parameters estimates of the time-varying quadratic asymmetric model. The robust t-stats are in parentheses. T-stats larger than two are in bold. Panel 2 shows Wald tests of significance and variability of covariance asymmetry.

<table>
<thead>
<tr>
<th>Panel 1</th>
<th>$\delta_{1,t}$</th>
<th>$\delta_{2,t}$</th>
<th>$\delta_{3,t}$</th>
<th>$\delta_{4,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_{c,1}$</td>
<td>0.027</td>
<td>0.005</td>
<td>-0.001</td>
<td>0.009</td>
</tr>
<tr>
<td>(2.082)</td>
<td>(0.251)</td>
<td>(-0.048)</td>
<td>(0.477)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{c,2}$</td>
<td>-0.041</td>
<td>-0.215</td>
<td>0.459</td>
<td>0.442</td>
</tr>
<tr>
<td>(-0.185)</td>
<td>(-0.728)</td>
<td>(1.452)</td>
<td>(1.483)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{c,3}$</td>
<td>-0.049</td>
<td>0.170</td>
<td>-0.041</td>
<td>0.020</td>
</tr>
<tr>
<td>(-0.651)</td>
<td>(1.841)</td>
<td>(-0.513)</td>
<td>(0.234)</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.030</td>
<td>0.129</td>
<td>0.098</td>
<td>0.102</td>
</tr>
<tr>
<td>(0.468)</td>
<td>(2.883)</td>
<td>(1.163)</td>
<td>(1.770)</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.178</td>
<td>-2.244</td>
<td>-1.335</td>
<td>-0.763</td>
</tr>
<tr>
<td>(-0.103)</td>
<td>(-2.638)</td>
<td>(-1.095)</td>
<td>(-0.014)</td>
<td></td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>0.127</td>
<td>0.003</td>
<td>0.323</td>
<td>-0.141</td>
</tr>
<tr>
<td>(0.439)</td>
<td>(0.013)</td>
<td>(0.866)</td>
<td>(-0.557)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{1,1}$</td>
<td>0.224</td>
<td>0.219</td>
<td>0.208</td>
<td>0.207</td>
</tr>
<tr>
<td>(11.511)</td>
<td>(17.199)</td>
<td>(12.002)</td>
<td>(16.270)</td>
<td></td>
</tr>
</tbody>
</table>

Panel 2

Do international returns display symmetric covariance?
$H_0$: The coefficients $\theta_i = 0$ for all $i$'s

Do international returns display constant asymmetric covariance?
$H_0$: The coefficients $\kappa_{i,2} = \kappa_{i,3} = 0$ for $i = 1,..., 8$

Do international equities display symmetric covariance?
$H_0$: The coefficients $\theta_i = 0$ for $i = 1,...,4$, and 8

Do international equities display constant symmetric covariance?
$H_0$: The coefficients $\kappa_{i,2} = \kappa_{i,3} = 0$ for $i = 1, 2, 3, 4$ and 8

Do foreign deposits display symmetric covariance?
$H_0$: The coefficients $\theta_i = 0$ for $i = 5, 6$, and 8

<table>
<thead>
<tr>
<th>$\chi^2$</th>
<th>p-val</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>94.93</td>
<td>0.000</td>
<td>24</td>
</tr>
<tr>
<td>25.54</td>
<td>0.061</td>
<td>16</td>
</tr>
<tr>
<td>75.43</td>
<td>0.000</td>
<td>15</td>
</tr>
<tr>
<td>19.05</td>
<td>0.040</td>
<td>10</td>
</tr>
<tr>
<td>3.18</td>
<td>0.957</td>
<td>9</td>
</tr>
</tbody>
</table>
Table IV
GMM test for the total risk premium differences. The table reports estimates of $\hat{\phi}_i$, with $i = 1, \ldots, 8$, for the 8 assets from the three GMM regressions 1) $RP_{i,t}^{GJR} - RP_{i,t}^{B} = \phi_i + \epsilon_t$, 2) $RP_{i,t}^{TVA} - RP_{i,t}^{B} = \phi_i + \epsilon_t$, and 3) $RP_{i,t}^{TVA} - RP_{i,t}^{GJR} = \phi_i + \epsilon_t$, where the superscript identifies the model used to compute the risk premium. B denotes the benchmark, GJR denotes the constant GJR asymmetric model, and TVA denotes the time-varying asymmetric model. The heading of each column indicates to which of the 8 assets the GMM risk premium test refers to. The heading of each row indicates which models are used to estimate the two risk premia that are compared. The t-stats are corrected for autocorrelation and heteroskedasticity with the Newey and West (1987) estimator with the Andrews (1991) automatic bandwidth selection. T-stats larger than two are in bold. The unit of measure is percentage per year. The risk premia estimated with time-varying asymmetry are considerably lower for equities than those estimated allowing for no asymmetry.

<table>
<thead>
<tr>
<th>RP Difference</th>
<th>GER</th>
<th>JAP</th>
<th>UK</th>
<th>US</th>
<th>DEM</th>
<th>JPY</th>
<th>GBP</th>
<th>WOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RP^{(GJR)} - RP^{(B)}$, $t - stats$</td>
<td>-0.62</td>
<td>-0.88</td>
<td>-0.74</td>
<td>-0.92</td>
<td>0.17</td>
<td>-0.25</td>
<td>-0.16</td>
<td>-0.89</td>
</tr>
<tr>
<td>$RP^{(TVA)} - RP^{(B)}$, $t - stats$</td>
<td>-1.09</td>
<td>-1.57</td>
<td>-0.79</td>
<td>-1.89</td>
<td>0.64</td>
<td>0.57</td>
<td>0.83</td>
<td>-1.58</td>
</tr>
<tr>
<td>$RP^{(TVA)} - RP^{(GJR)}$, $t - stats$</td>
<td>-0.47</td>
<td>-0.70</td>
<td>-0.05</td>
<td>-0.97</td>
<td>0.47</td>
<td>0.82</td>
<td>0.98</td>
<td>-0.69</td>
</tr>
</tbody>
</table>
Figure 1
Total risk premium. The plots show the total risk premium in percentage per year for each of the 8 assets estimated using the benchmark, the GJR, and the model with time varying asymmetry. The pictures address visually the question formally examined in the GMM test. It appears that the estimated risk premium commanded by equities is lower when the benchmark (symmetric model) is used than it is when the same premium is estimated with the time varying asymmetric model. For the foreign exchange risk premia the plots often show the opposite pattern, i.e. the risk premium for foreign exchange is higher when it is estimated with the model featuring time varying asymmetry than it is when it is estimated allowing for no asymmetry. The series are exponentially smoothed to eliminate high order components (smoothing parameter .02).
Figure 2
The plot shows the price of the market risk $\delta_{1,t}$, the price of DEM risk $\delta_{2,t}$, the price of JPY risk $\delta_{3,t}$, and the price of GBP risk $\delta_{4,t}$ estimated from the benchmark, the JGR asymmetric model with market spillovers, and the model with time varying asymmetry. To eliminate high frequency components, the prices of risk were smoothed with a simple exponential filter (smoothing parameter 0.02). The shaded areas start at the peak of each business cycle and end at the trough according to the dates from NBER.