Sample First Midterm Exam

Last Name: ________________________________
First Name: ________________________________
Student ID Number: _________________________

Preliminaries

Exam time is: 80 minutes.
Total points for this exam is:  400 points

This is a closed book and notes exam. One hand written formula sheet is allowed. (No photocopy or print out). Remove all other paper from your desk.

Explain carefully when answering each question. Show all relevant calculations in the space provided under each question. Use the back of each page for extra space and for scrap paper. Simply showing solution, even if correct, may not give you full credit.

All exams must be turned in the day of the exam and during the class graded exams are reviewed.

This exam is subject to KSU’s Code of Conduct, as published in the Undergraduate and Graduate Catalogs. Violators will be prosecuted.
SAMPLE QUESTIONS

Question (50 points)

What is the price of a 5-year bond with a nominal value of $100, a yield to maturity of 7% (with annual compounding frequency), a 12% coupon rate and a semi annual coupon frequency?

<table>
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<th>DF</th>
<th>DCF</th>
</tr>
</thead>
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<td>0.97</td>
<td>5.80</td>
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<td>6</td>
<td>0.93</td>
<td>5.60</td>
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<td>6</td>
<td>0.90</td>
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<td>6</td>
<td>0.87</td>
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<td>4.40</td>
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<td>10</td>
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<td>0.71</td>
<td>75.15</td>
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<td>120.79151</td>
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</tbody>
</table>

Question (50 points)

An investor has a cash of $10,000,000 at disposal. He wants to invest in a bond with $1,000 nominal value and whose dirty price is equal to 105%.

1. What is the number of bonds he will buy?
2. Same question if the nominal value and the dirty price of the bond are respectively $100 and 97%.

Solution

1. The number of bonds he will buy is given by the following formula
   Number of bonds bought = Cash / (Nominal Value of the bond × dirty price)

Here, the number of bonds is equal to

n = 10000000 / (1000 * 105%) = 9523.81
and
n = 10,000,000 / (100 * 97%) = 103093
**Question (50 points)**

On 05/31/2002, an investor buys $1 million US T-Bill with maturity date 06/27/2002 and discount yield 1.76% on the settlement date. (Recall that the settlement date is the next business day and the day convention is 360.)

1. What is the price of the T-Bill?
2. What is the equivalent money-market yield?

**Solution**

The settlement date of the transaction is 06/1/2002 (trading date plus 1 working day). There are 26 calendar days between the settlement date and the maturity date.

The price $P$ of the T-Bill is equal to

\[ 100 \times (1 - 1.76\% \times 26/360) = 99.8729 \]

2. The equivalent money-market yield is equal to

\[ (1.76\% \times 360)/(360 - 26 \times 1.76\%) = 1.76224\% \]

**Question (50 points)**

Show that the price of a consol bond (perpetuity) is

\[ P = cN/r \]

Without loss of generality assume $cN = 1$

\[ P = (1+r)^{-1} + (1+r)^{-2} + (1+r)^{-3} + (1+r)^{-4} \ldots \]

Hence

\[ P(1+r) = 1 + (1+r)^{-1} + (1+r)^{-2} + (1+r)^{-3} + (1+r)^{-4} \ldots \]

Subtract the first equation from the second

\[ P(1+r) - P = 1 \]

Which implies $P = 1/r$, or $P = cN/r$ under the assumption that $cN = 1$. 

Question (50 points)

If an investment has a cumulative 63.45% rate of return over 3.78 years, what is the annual continuously compounded rate of return?

Solution. The annual continuously compounded rate of return $R$ is such that

$$1.6345 = \exp(3.78R_c)$$

We find $R_c = \ln(1.6345)/3.78 = 13\%$. 
**Question (50 points)**

How would you compute the yield to maturity of a bond that matures in 6 years, has a coupon rate of 4% paid annually, a face value of 1000, and a price equal to 970?

a) Write the equation
b) Use the space below to write the cash flows, discount factor, and discounted cash flows for an arbitrary rate of 5%.
c) Compute the price with your financial calculator
d) Explain how you would find the yield solving the appropriate equality in excel
e) Compare the price you computed with the market price. Is the yield higher or lower? Can you compute it with your financial calculator?

\[
970 = \frac{40}{1+y} + \frac{40}{(1+y)^2} + \cdots + \frac{1040}{(1+y)^6}
\]

Solve for y.

<table>
<thead>
<tr>
<th>time</th>
<th>CF</th>
<th>DF</th>
<th>DCF</th>
<th>price</th>
<th>Difference</th>
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Instruct the solver to make the sum of discounted cash flows equal to the price

Rate is 0.045832
Question (50 points)

What are the three main components affecting the change in the yield curve? Sketch them in three different plots.

Answer.
Check your slides.
Question (50 points)
What does the “pure expectation theory” of the yield curve say?

What does the “Risk premium theory” of the yield curve say?

What does the “Preferred habitat theory” of the yield curve say?

How does the Pure Risk premium theory explain a hump shaped yield curve?
Answer. It doesn’t.
**Question (50 points)**
Explain the basic difference that exists between the preferred habitat theory and the segmentation theory.

Answer

In the segmentation theory, investors are supposed to be 100% risk-averse. Therefore, risk premia are infinite. It is as if their investment habitat were strictly constrained, exclusive.

In the preferred habitat theory, investors are not supposed to be 100% risk averse. So, there exists a certain level of risk premia from which they are ready to change their habitual investment maturity. Their investment habitat is, in this case, not exclusive.
Question 7 (50 points)

At date $t = 0$, we observe the following zero-coupon rates in the market:

<table>
<thead>
<tr>
<th>Maturity Zero-Coupon</th>
<th>Years</th>
<th>Rate</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
<td>5.0%</td>
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<tr>
<td></td>
<td>2</td>
<td>6.0%</td>
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<tr>
<td></td>
<td>3</td>
<td>6.5%</td>
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<tr>
<td></td>
<td>4</td>
<td>6.8%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7.0%</td>
</tr>
</tbody>
</table>

a) Draw the yield curve
b) What are the 1-year maturity forward rates $F(0, t, 1)$ implied by the current term structure?
c) Draw the Forward rate and the zero curve on top of each other.
Question 8 (50 points)

Show that

\[ r_{BEY} \times P \times \frac{360}{365} = r_{BD} \]

Where \( r_{BEY} \) is the Bond equivalent yield, \( r_{BD} \) is the bank discount yield and \( P \) is the price of a bond with face value equal to $1.

Solution

\[ P = 1 - r_{BD} \frac{n}{360} \]

\[ P = \frac{1}{1 + r_{BEY} \frac{n}{365}} \]

\[ \Rightarrow \]

\[ \frac{1}{1 + r_{BEY} \frac{n}{365}} = 1 - r_{BD} \frac{n}{360} \]

Solve for \( r_{BD} \)

\[ 1 - \frac{1}{1 + r_{BEY} \frac{n}{365}} = r_{BD} \frac{n}{360} \]

\[ \frac{1 + r_{BEY} \frac{n}{365}}{1 + r_{BEY} \frac{n}{365}} - 1 \cdot \frac{360}{n} = r_{BD} \]

\[ r_{BEY} \frac{1}{1 + r_{BEY} \frac{n}{365}} \frac{360}{n} = r_{BD} \]

\[ r_{BEY} \frac{360}{365} = r_{BD} \]