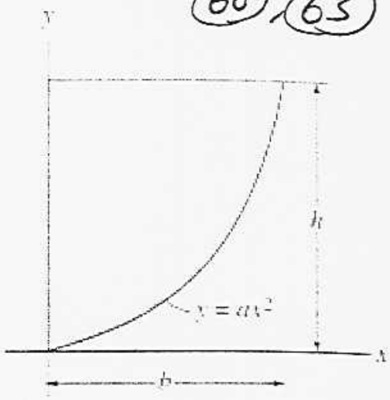
HW # 10 Solutions.

ch 9 : 7, 10, 16, 18, 28, 57, 58, 60, 63

$\boxed{9-7}$



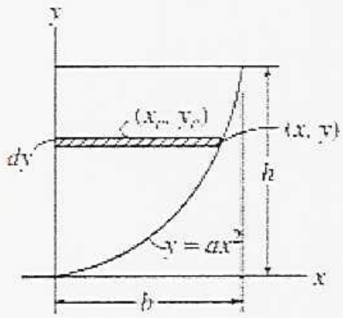
$$dA = x\ dy$$

$$x_C = \frac{x}{2}$$

$$y_C = y$$

$$A = \int_0^h b\sqrt{\frac{y}{h}}\ dy = h\ b\left(\frac{y}{h}\right)^{\frac{1}{2}}$$

$$x_C = \frac{3}{2hb}\int_0^h \frac{1}{2}\left(b\sqrt{\frac{y}{h}}\right)^2 dy = \frac{3}{8\ h^2}\ b\ h^2$$



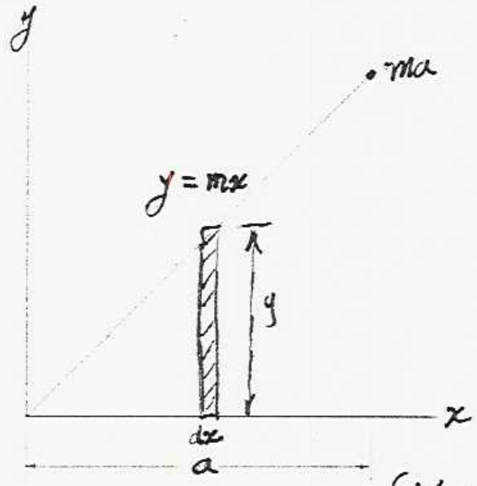
$$x_C = \frac{3}{8}b$$

$$y_C = \frac{3}{2hb}\int_0^h yb\sqrt{\frac{y}{h}}\ dy = \frac{3}{5}\ h\left(\frac{h}{h}\right)^{\frac{5}{2}}$$

$$y_C = \frac{3}{5}h$$

Problem 9-10

Determine the location (x_c, y_c) of the centroid of the triangular area.



$dA = y\,dx$

$C: \left(x, \frac{y}{2}\right)$

$A = \frac{1}{2} m a^2$

$\begin{cases} \text{if } x = a \\ y = ma \end{cases}$

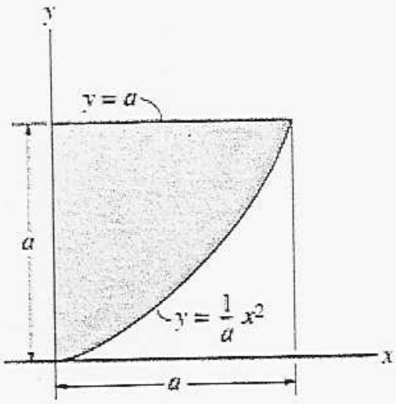
Solution:

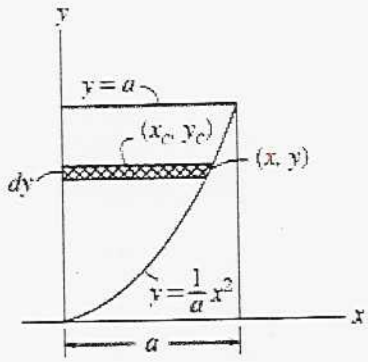
$$A = \int_0^a m x \, dx = \frac{1}{2} a^2 m$$

$$x_C = \frac{2}{m a^2} \int_0^a x m x \, dx = \frac{2}{3} a \qquad x_C = \frac{2}{3} a$$

$$y_C = \frac{2}{m a^2} \int_0^a \frac{1}{2} (m x)^2 \, dx = \frac{1}{3} a\, m \qquad y_C = \frac{m}{3} a$$

9-16





Solution:

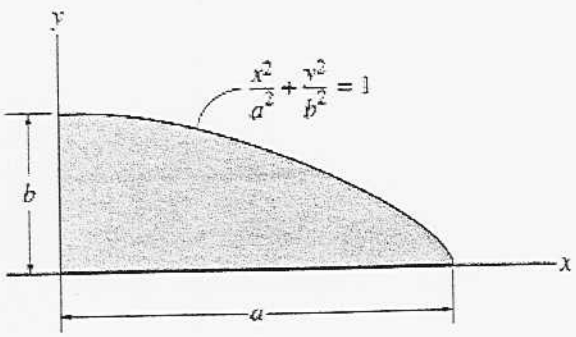
$$A = \int_0^a \sqrt{a\,y}\,dy = \frac{2}{3}\,\frac{\left(a^2\right)^{\frac{3}{2}}}{a} \qquad A = \frac{2a^2}{3}$$

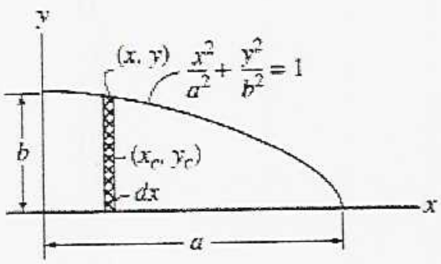
$$x_c = \frac{3}{2a^2}\int_0^a \frac{1}{2}a\,y\,dy = \frac{3}{8}\,a \qquad x_c = \frac{3}{8}a$$

$$y_c = \frac{3}{2a^2}\int_0^a y\sqrt{a\,y}\,dy = \frac{3}{5\,a^4}\left(a^2\right)^{\frac{5}{2}} \qquad y_c = \frac{3}{5}a$$

Problem 9-17

Locate the centroid of the quarter elliptical area.





Solution:

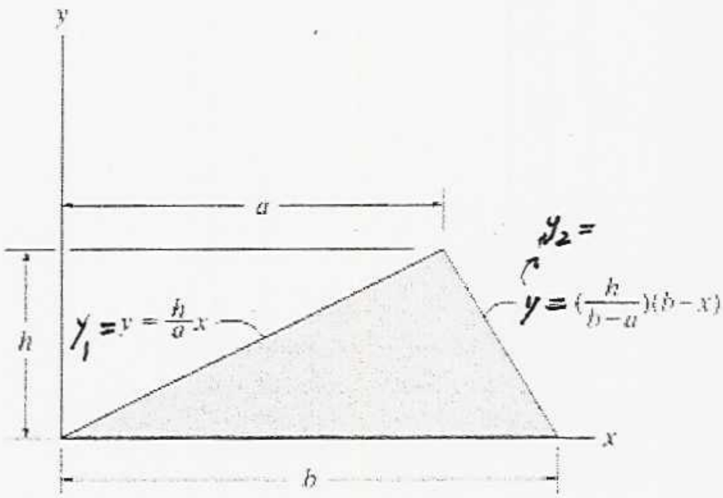
$$A = \int_0^a b\sqrt{1 - \left(\frac{x}{a}\right)^2}\,dx \qquad\qquad A = \frac{\pi a b}{4}$$

$$x_C = \frac{4}{\pi a b}\int_0^a x b\sqrt{1 - \left(\frac{x}{a}\right)^2}\,dx = \frac{4}{3\,\pi}\,a \qquad\qquad x_C = \frac{4}{3\pi}a$$

$$y_C = \frac{4}{\pi a b}\int_0^a \frac{1}{2}\left[b\sqrt{1 - \left(\frac{x}{a}\right)^2}\right]^2 dx = \frac{4}{3\,\pi}\,b \qquad\qquad y_C = \frac{4}{3\pi}b$$

Problem 9-18

Locate the centroid x_c of the triangular area.



Solution:

$$A = \frac{b h}{2}$$

$$x_C = \frac{2}{b h}\left[\int_0^a x\overbrace{\frac{h}{a}x}^{y_1}\,dx + \int_a^b x\overbrace{\frac{h}{b-a}(b-x)}^{y_2}\,dx\right] \qquad\qquad x_C = \frac{a+b}{3}$$

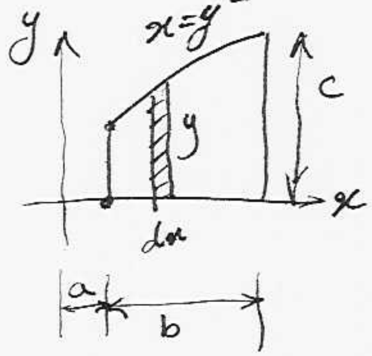
general form of equation $\left(\frac{y}{c}\right)^2 = \frac{x}{a+b}$

$\boxed{9\text{-}28}$

Solution:

$dA = y\,dx$

$\tilde{y} = \frac{y}{2}$



a = 1"
b = 3"
c = 2"

$$A = \int_a^{a+b} c\sqrt{\frac{x}{a+b}}\,dx$$

$$y_c = \frac{1}{A}\int_a^{a+b} \frac{1}{2}\left(c\sqrt{\frac{x}{a+b}}\right)^2 dx \qquad y_c = 0.804\text{ in}$$

Problem 9-29

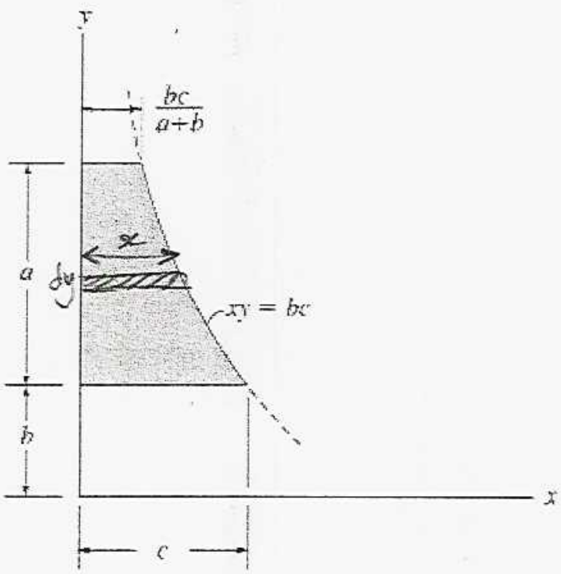
Locate the centroid x_c of the shaded area.

Given:

$a = 4$ in

$b = 2$ in

$c = 3$ in



Solution:

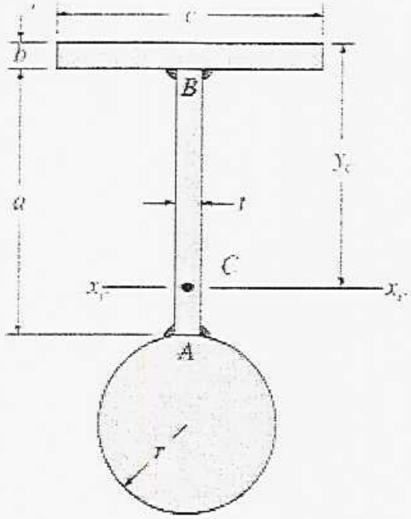
$$A = \int_b^{a+b} \frac{bc}{y}\,dy \qquad A = 6.592\text{ in}^2$$

$$x_c = \frac{1}{A}\int_b^{a+b} \frac{1}{2}\left(\frac{bc}{y}\right)^2 dy \qquad x_c = 0.910\text{ in}$$

Problem 9-57

Determine the location y_c of the centroidal axis $x_c x_c$ of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.

Given:



$r = 50$ mm

$t = 15$ mm

$a = 150$ mm

$b = 15$ mm

$c = 150$ mm

Solution:

$$y_c = \frac{bc\left(\frac{b}{2}\right) + at\left(b + \frac{a}{2}\right) + \pi r^2 (b + a + r)}{bc + at + \pi r^2} \qquad y_c = 154.443 \text{ mm}$$

Problem 9-58

Determine the location (x_c, y_c) of the centroid C of the area.

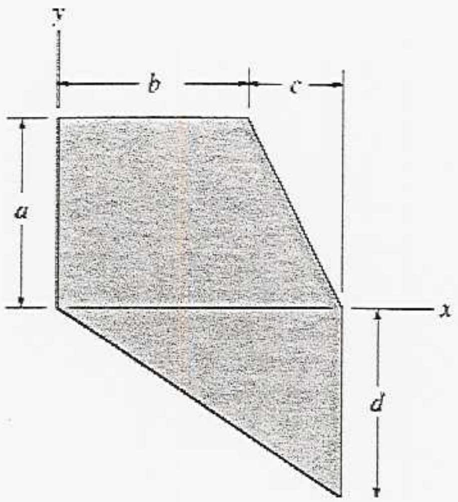
Given:

$a = 6$ in

$b = 6$ in

$c = 3\ \text{in}$

$d = 6\ \text{in}$



Solution:

$$x_c = \frac{ab\left(\frac{b}{2}\right) + \frac{1}{2}ac\left(b + \frac{c}{3}\right) + \frac{1}{2}(b+c)d\frac{2}{3}(b+c)}{ab + \frac{1}{2}ca + \frac{1}{2}(b+c)d} \qquad x_c = 4.625\ \text{in}$$

$$y_c = \frac{ab\left(\frac{a}{2}\right) + \frac{1}{2}ac\left(\frac{a}{3}\right) - \frac{1}{2}(b+c)d\left(\frac{d}{3}\right)}{ab + \frac{1}{2}ca + \frac{1}{2}(b+c)d} \qquad y_c = 1\ \text{in}$$

Problem 9-59

Determine the location y_c of the centroid C for a beam having the cross-sectional area shown. The beam is symmetric with respect to the y axis.

Given:

$a = 2\ \text{in}$

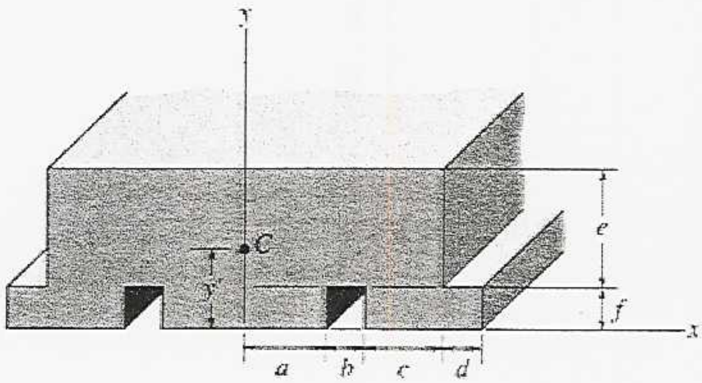
$b = 1\ \text{in}$

$c = 2\ \text{in}$

$d = 1\ \text{in}$

$e = 3\ \text{in}$

$f = 1\ \text{in}$



Solution:

$$A = 2[(a+b+c+d)(e+f) - bf - de] \qquad A = 40\ \text{in}^2$$

$$y_C = \frac{2}{A}\left[(a+b+c+d)\frac{(e+f)^2}{2} - b\frac{f^2}{2} - de\left(f+\frac{e}{2}\right)\right] \qquad y_C = 2.00\text{ in}$$

Problem 9-60

The wooden table is made from a square board having weight W. Each of the legs has wieght W_{leg} and length L. Determine how high its center of gravity is from the floor. Also, what is the angle, measured from the horizontal, through which its top surface can be tilted on two of its legs before it begins to overturn? Neglect the thickness of each leg.

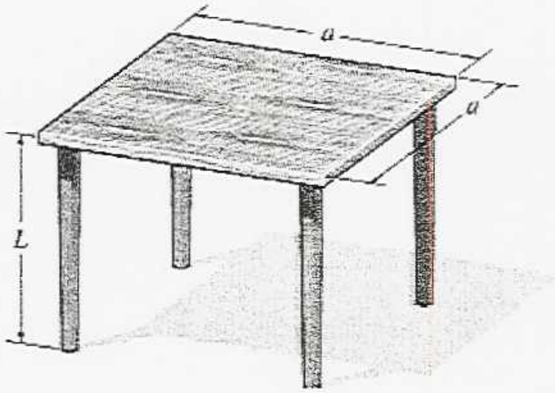
Given:

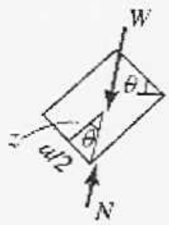
$W = 15$ lb

$W_{leg} = 2$ lb

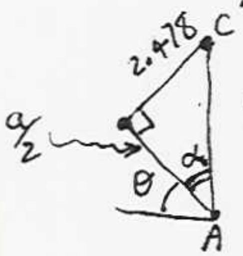
$L = 3$ ft

$a = 4$ ft





center of gravity



$\tan\alpha = \frac{2.478}{2}$

$\alpha = 51.1°$

Solution:

$$z_C = \frac{WL + 4W_{leg}\left(\frac{L}{2}\right)}{W + 4W_{leg}} \qquad z_C = 2.478\text{ ft}$$

$$\theta = \operatorname{atan}\left(\frac{\frac{a}{2}}{z_C}\right) \qquad \theta = 38.9\text{ deg}$$

So: $\theta = 38.9°$

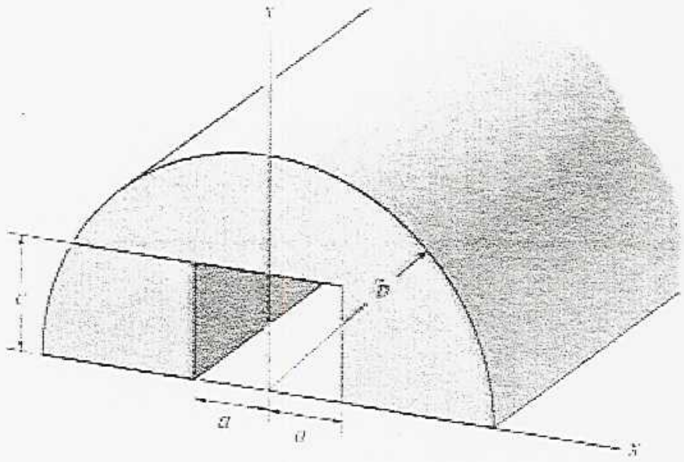
Problem 9-61

Locate the centroid y_c for the beam s cross-sectional area.

Given:

$a = 120$ mm

Problem 9-63



Locate the centroid y_c for the strut s cross-sectional area.

Given:

$$a = 40\text{ mm}$$

$$b = 120\text{ mm}$$

$$c = 60\text{ mm}$$

Solution:

$$A = \frac{\pi b^2}{2} - 2ac$$

$$y_c = \frac{1}{A}\left[\frac{\pi b^2}{2}\left(\frac{4b}{3\pi}\right) - 2ac\left(\frac{c}{2}\right)\right] \qquad y_c = 56.6\text{ mm}$$