

4-135 Solution:

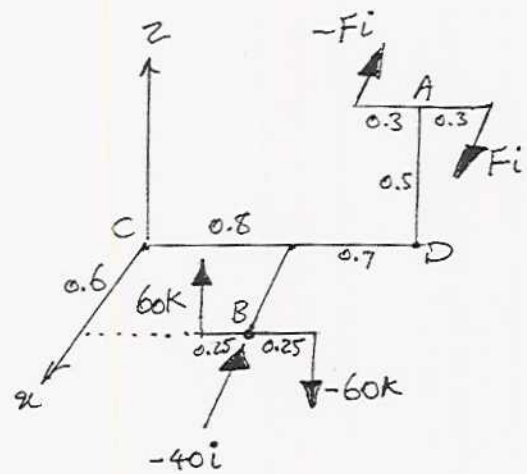
Find the resultant force:

$$\vec{F}_R = -40\hat{i} + 60\hat{k} - 60\hat{k} + F\hat{i} - F\hat{i}$$

(couple forces are cancelled out)

$$\vec{F}_R = -40\hat{i}$$

The resultant force is along  $x$  direction (negative  $x$ ) and for having a wrench, the resultant moment should be along  $x$  axis as well.



Units are in N and m.

Find all the moments due to couple forces and force  $(-40\hat{i})$ :

$$\vec{M}_{total} = \vec{r}_{CB} \times (-40\hat{i}) + \ominus(60 \times 0.5)\hat{i} + \ominus(F \times 0.6)\hat{k}$$

$$\vec{M}_{total} = (0.6\hat{i} + 0.8\hat{j}) \times (-40\hat{i}) - 30\hat{i} - 0.6F\hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}, \hat{i} \times \hat{i} = 0$$

$$\vec{M}_{total} = 32\hat{k} - 30\hat{i} - 0.6F\hat{k} = -30\hat{i} + (32 - 0.6F)\hat{k}$$

$$\vec{M}_{total} = \vec{M}_R = |M_R|\hat{i} \quad \left( \begin{array}{l} \text{the only conclusion we get from} \\ \text{having a wrench} \end{array} \right)$$

$$-30\hat{i} + (32 - 0.6F)\hat{k} = |M_R|\hat{i} \quad \text{corresponding components are equal}$$

$$-30 = |M_R| \text{ N.m}$$

Resultant moment is along negative  $x$

$$32 - 0.6F = 0 \Rightarrow F = \frac{32}{0.6} = 53.3 \text{ N}$$

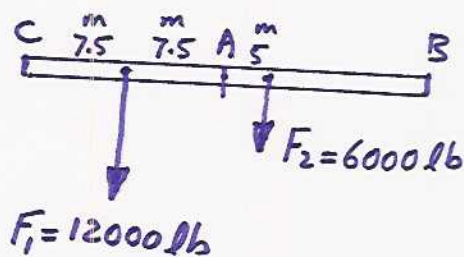
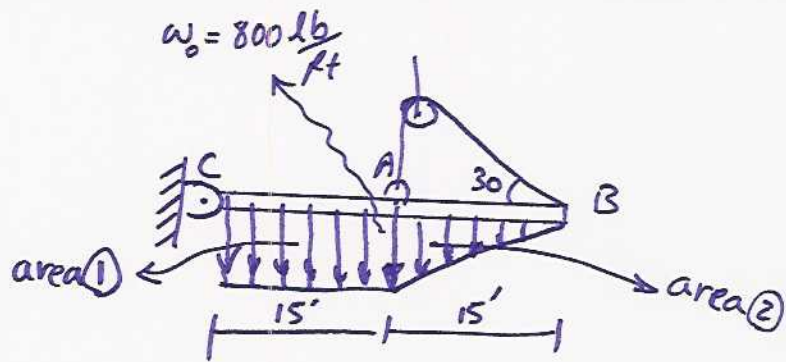
4-145

$F_1$  = Resultant force for rectangular distributed load

$$= \text{area 1} = 15 \times 800 = 12000 \text{ lb} = 12 \text{ kip}$$

$F_2$  = Resultant force for triangular distributed load

$$= \text{area 2} = \frac{800 \times 15}{2} = 6000 \text{ lb} = 6 \text{ kip}$$



$$F_R = F_1 + F_2 = 12000 \text{ lb} + 6000 \text{ lb} = 18000 \text{ lb} = 18 \text{ kip}$$

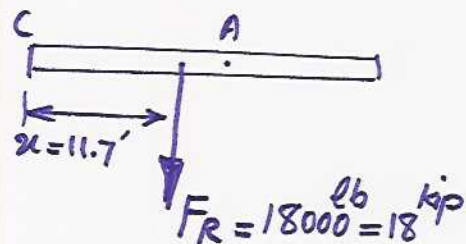
Centroid of area 1 is 7.5' away from C. ( $= \frac{1}{2} AC$ )

Centroid of area 2 is 20' away from C ( $= AC + \frac{1}{3} AB$ )

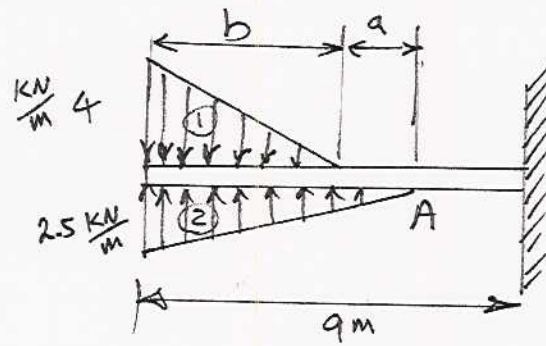
$$M_C = -F_1(7.5) - F_2(20) = -12 \times 7.5 - 6 \times 20 = -210 \text{ kip ft}$$

$$M_R = -F_R \cdot x = -18 \cdot x$$

$$M_C = M_R = -18x = -210 \rightarrow x = \frac{210}{18} = 11.7 \text{ ft}$$



**Problem 4-147**



$$F_1 = \text{area } \textcircled{1}$$

$$F_2 = \text{area } \textcircled{2}$$

$$F_1 = \frac{4 \times b}{2} = 2b$$

$$F_2 = \frac{2.5 \times (a+b)}{2} = \frac{5}{4}(a+b)$$

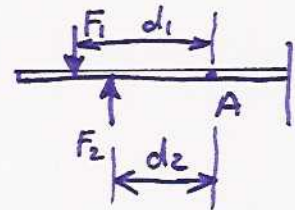
Since  $F_R = 0 \Rightarrow F_1 = F_2 \rightarrow 2b = \frac{5}{4}(a+b) \Rightarrow \boxed{b = \frac{5}{3}a}$

$\sum M_A = 8 \text{ kN.m}$  (the book should read counter-clockwise)

This is a couple moment (since two forces are equal) and you can consider it about any point.

$$\sum M_A = F_1 \left( \frac{2}{3}b + a \right) - F_2 \left( \frac{2}{3}(a+b) \right) = 8 \text{ kN.m}$$

OR  $\sum M_A = F_1 \cdot d_1 - F_2 \cdot d_2 = 8 \text{ kN.m}$



substituting  $F_1$  &  $F_2$ :

$$2b \left( \frac{2}{3}b + a \right) - 1.25(a+b) \left( \frac{2}{3}(a+b) \right) = 8$$

$$\frac{4}{3}b^2 + 2ab - (1.25a + 1.25b) \left( \frac{2}{3}a + \frac{2}{3}b \right) = 8$$

$$b = \frac{5}{3}a \Rightarrow 1.12 a^2 = 8 \xrightarrow{\text{kN.m}} a = 2.682^{\text{m}}, \quad b = 4.47^{\text{m}}$$

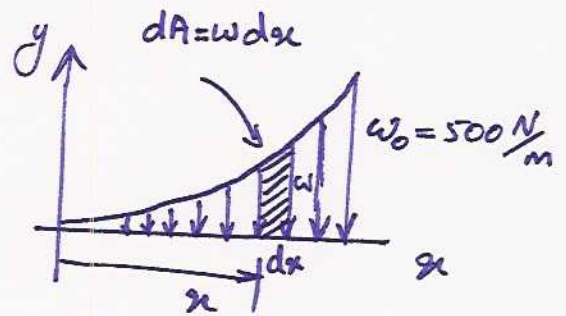
Problem 4-156

$$F_R = \int_0^{10} \omega dx$$
$$= \int_0^{10} \frac{1}{2} x^3 dx = \frac{x^4}{8} \Big|_0^{10}$$

$$F_R = \frac{10^4}{8} - 0 = 1250 \text{ N}$$

$$\bar{x} = \frac{\int_L x \omega(x) dx}{\int_L \omega(x) dx} = \frac{\int_0^{10} x \cdot (\frac{1}{2} x^3) dx}{1250} = \frac{1}{1250} \left( \frac{1}{2} \cdot \frac{x^5}{5} \right) \Big|_0^{10} = \frac{x^5}{12500} \Big|_0^{10} = 8 \text{ m}$$

↑  
this is  $F_R$



$$\omega(x) = \frac{1}{2} x^3 \frac{\text{N}}{\text{m}}$$

