The Mathematics 11
Competency Test

More Unit Conversion Examples

In this document, we present a few more examples of unit conversions, now involving units of measurement both in the SI and others which are not in the SI. The purpose of these additional examples is to illustrate further the general strategy presented in the previous document.

UNIT CONVERSION FACTORS

First, we must give you a very brief table of some unit conversion factors for common physical quantities. This is a very abbreviated list of unit conversion factors. There are many references or handbooks giving much more extensive listings. One is the *CRC Handbook of Chemistry and Physics*, updated annually by the Chemical Rubber Company. Another is *The Metric Guide*, published by the Council of Ministers of Education, Canada. You can also find extensive listings of unit conversion factors on the internet. All of the factors given in **bold** below are exact either by definition or by international convention, according to this second reference.

LENGTH
- 1 in = **2.54 cm**
- 1 ft = **12 in** = **30.48 cm**
- 1 yd = **3 ft**
- 1 mi = **5280 ft**
- 1 nautical mile = **6080 ft**
- 1 mi = **1.609344 km**

AREA
- 1 mi^2 = **640 acres**
- 1 hectare = **10000 m^2** = **(100 m)^2**

VOLUME
- 1 imp. gal = **4.546090 litres** = **4 quarts**
- 1 imp. quart = **40 fluid ounces**
- 1 bushel = **8 gallons**
- 1 litre = **1000 cm^3**
- 1 U. S. gal (liq) = 3.785306 litres
- 1 Hogshead = 238.47427 litres
- 1 Noggin = 142.0652 cm^3

WEIGHT/MASS
- 1 lb = **16 oz**
- 1 lb = **453.59237 g**
- 1 ton = **2000 lb**
- 1 metric ton = **1000 kg**
- 1 carat = 200 mg
- 1 slug = 14.5939 kg

ENERGY
- 1 Btu = 1055.6 J (N m = kg m^2/s^2)
- 1 calorie = **4.1868 J**
- 1 Joule = **10^7 ergs** (g cm^2/s^2)

FORCE
- 1 Newton = 0.22480894 lb
- 1 Newton = **10^5 dynes** (g cm/s^2)

POWER
- 1 horsepower (mechanical)
  = 550 ft lb/s = 745.700 watts

NOTES: The following abbreviations are used in the table above:

ft = foot	yd = yard	mi = mile
oz = ounce	lb = pound	imp gal = imperial gallon

There is a fairly fascinating history to the development of the hundreds of non-SI units that have been used in various geographical areas and societies in recent centuries (see http://www.unc.edu/~rowlett/units/custom.html for one account. In particular, as reflected in the table above, the standardization of the 'gallon,' a unit of volume was done differently by the United States and the British, resulting in a U. S. gallon which is somewhat smaller than the British 'imperial gallon.' Furthermore, the U. S. distinguishes between a gallon for measuring volume of liquids such as wine and a gallon for measure volume of dry goods such as grain.

Example 1: Convert the length 56.43 ft to its equivalent in units of metres.

solution:
We have

1 ft = 30.48 cm

and

1 m = 100 cm

So, an effective strategy would appear to be

ft → cm → m

The required template is

$$56.43\ ft = 56.43\ ft \left(\frac{cm}{ft}\right)\left(\frac{m}{cm}\right)$$

Filling in the numbers and doing the arithmetic gives:

$$56.43\ ft = 56.43\ \cancel{ft} \left(\frac{30.48\ \cancel{cm}}{1\ \cancel{ft}}\right)\left(\frac{1\,m}{100\ \cancel{cm}}\right)$$

$$= \frac{(56.43)(30.48)}{100}\ m$$

$$= 17.199864\ m$$

Thus, rounding to two decimal places, we can state that

56.43 ft = 17.20 m

as the final result.

Example 2: Highway regulations in a certain jurisdiction state that a truck with two axles may have a maximum weight of 16000 pounds on the front axle and 20000 pounds on the rear axle. Convert these weights to their equivalents in kilograms, rounded to the nearest ten kilograms.

solution:
From our table of unit conversion factors, we have

1 pound = 1 lb = 453.59237 g

and we know that

1 kg = 1000 g

So, an effective strategy would appear to be

lb → g → kg

The template for converting the maximum front axle weight is

$$16000\ lb = 16000\ lb\left(\frac{g}{lb}\right)\left(\frac{kg}{g}\right)$$

Filling in the numbers and doing the arithmetic gives

$$16000\ lb = 16000\ \cancel{lb}\left(\frac{453.59237\ \cancel{g}}{1\ \cancel{lb}}\right)\left(\frac{1\ kg}{1000\ \cancel{g}}\right)$$

$$= \frac{(16000)(453.59237)}{1000}\ kg = 7257.48\ kg$$

The calculation for the maximum rear axle weight goes exactly the same way:

$$20000\ lb = 0000\ \cancel{lb}\left(\frac{453.59237\ \cancel{g}}{1\ \cancel{lb}}\right)\left(\frac{1\ kg}{1000\ \cancel{g}}\right)$$

$$= 9071.85\ kg$$

Thus, rounding to the nearest 10 kg, we can stated that these regulations correspond to a maximum of 7260 kg on the front axle, and 9070 kg on the rear axle.

Example 3: According to one internet site, the distance from Vancouver, B. C. to Hong Kong is 5550 nautical miles, as the crow flies. Ignoring the question of whether any crow really could fly that far, determine what this distance is in units of kilometres.

solution:
From our list of unit conversion factors, we have that

1 nautical mile = 6080 feet

and

1 foot = 30.48 cm

and we can easily convert centimetres to kilometres. So, an effective strategy here would appear to be

naut. mi $\rightarrow$ ft $\rightarrow$ cm $\rightarrow$ m $\rightarrow$ km

The template for this calculation is

$$5550\ naut.mi = 5550\ naut.mi\left(\frac{ft}{naut.mi}\right)\left(\frac{cm}{ft}\right)\left(\frac{m}{cm}\right)\left(\frac{km}{m}\right)$$

Filling in the numbers and completing the calculation gives

$$5550\ naut.mi = 5550\ \cancel{naut.mi}\left(\frac{6080\ \cancel{ft}}{1\ \cancel{naut.mi}}\right)\left(\frac{30.48\ \cancel{cm}}{1\ \cancel{ft}}\right)\left(\frac{1\ \cancel{m}}{100\ \cancel{cm}}\right)\left(\frac{1\ km}{1000\ \cancel{m}}\right)$$

$$= \frac{(5550)(6080)(30.48)}{(100)(1000)}\ km$$

$$= 10285.1712\ km$$

Since the original length, 5550 nautical miles, appears to have just three significant figures, we should probably round our final stated answer to three significant figures as well. So, this distance between Vancouver, B. C. and Hong Kong is 10300 km.

Example 4: The speed limit on the Coquihalla Highway in British Columbia is 110 km/h. What is the equivalent speed in units of mi/h, rounded down to the nearest multiple of 5?

solution:
The easiest approach here is to convert the speed 110 km/h to its equivalent in mi/h, and then round the result appropriately. Since the time units are the same in both cases, we need only deal with the conversion between kilometres and miles. From our table of unit conversion factors, we get that

1 mi = 1.609344 km

So, it would appear that this problem can be done with the one step strategy

km $\rightarrow$ mi

The template for the calculation is

$$110\ \frac{km}{h} = 110\ \frac{km}{h}\left(\frac{mi}{km}\right)$$

Putting in the numbers and doing the calculations gives

$$110\ \frac{km}{h} = 110\ \frac{\cancel{km}}{h}\left(\frac{1\ mi}{1.609344\ \cancel{km}}\right)$$

$$= \frac{110}{1.609344}\ \frac{mi}{h} = 68.3508\ \frac{mi}{h}$$

Rounding down to the nearest multiple of 5 give the final answer of 65 mi/h.

Example 5: According to the U. S. Department of Energy, the 2003 model of the Chevrolet Malibu has a highway fuel economy rating of 29 mi/gal. Assuming this is a reference to the U. S.

gallon, compute the equivalent value of this fuel consumption rate in units of km/L. Round the answer to one decimal place.

solution:
Obviously this conversion will involve two parts:

mi $\rightarrow$ km

and

U. S. gal (liq) $\rightarrow$ litres

Our list of unit conversion factors gives numbers for both of these conversions, so these two lines form an effective strategy for this problem. The template for the calculation is

$$29\,\frac{mi}{gal} = 29\,\frac{mi}{gal}\left(\frac{\quad km}{mi}\right)\left(\frac{\quad gal}{L}\right)$$

Here, we've use the simple 'gal' to stand for 'U. S. gal (liq).' Putting the numbers into the template and doing the arithmetic then gives

$$29\,\frac{mi}{gal} = 29\,\frac{\cancel{mi}}{\cancel{gal}}\left(\frac{1.609344\ km}{1\ \cancel{mi}}\right)\left(\frac{1\ \cancel{gal}}{3.785306\ L}\right)$$

$$= \frac{(29)(1.609344)}{3.785306}\,\frac{km}{L} = 12.3295\,\frac{km}{L}$$

Rounded to one decimal place then, the requested fuel consumption rating is 12.3 km/L.

<u>Example 6</u>: A particularly fine variety of cheese is sold for \$1.47 per ounce. What is this price in dollars per kilogram?

solution:
What we need to do here is to convert ounces to kilograms, since the other unit, dollars, is the same in both the original value and the requested value. From the table of unit conversion factors, we have

1 pound = 1 lb = 16 oz = 16 ounces

and

1 lb = 453.59237 g

and we know how to convert from grams to kilograms. Thus, an effective strategy appears to be

oz $\rightarrow$ lb $\rightarrow$ g $\rightarrow$ kg

The template for this calculation is

$$1.47\,\frac{\$}{oz} = 1.47\,\frac{\$}{oz}\left(\frac{\quad oz}{lb}\right)\left(\frac{\quad lb}{g}\right)\left(\frac{\quad g}{kg}\right)$$

Putting the numbers into this template and then doing the arithmetic gives

$$1.47\,\frac{\$}{oz} = 1.47\,\frac{\$}{\not{oz}}\left(\frac{16\ \not{oz}}{1\ \not{lb}}\right)\left(\frac{1\ \not{lb}}{453.59237\ \not{g}}\right)\left(\frac{1000\ \not{g}}{1\,kg}\right)$$

$$= \frac{(1.47)(16)(1000)}{453.59237}\,\frac{\$}{kg} = 51.8527\,\frac{\$}{kg}$$

Since amounts of money are normally rounded to two decimal places, we state our final answer as: the price of this cheese is $51.85 per kilogram.

<u>Example 7</u>: Horse race lengths are often measured in furlongs, where 8 furlongs is equivalent to one mile. A fairly fast horse can run a 9 furlong race in about 1 minute and 45 seconds. Compute the speed of this horse in km/h.

solution:
Before attacking the main problem, we need to recognize that the time interval stated here has mixed units, minutes and seconds, and we must convert it to one of these units alone, or the other alone. For now, the easiest is to convert to seconds:

$$\text{1 minute 48 seconds} = \text{1 min} \times \frac{\text{60 s}}{\text{1 min}} + \text{48 s} = \text{108 s}$$

Thus, the horse's speed can initially be written as

$$\text{9 furlongs}/\text{108 s} \qquad or \qquad \frac{9}{108}\,\frac{\text{furlongs}}{\text{s}}$$

Now we can set up the required unit conversion calculation. Since

1 mi = 8 furlongs
1 mi = 1.609344 km
1 h = 60 min

and

1 min = 60 s,

then the two required stages of the unit conversion calculation can be accomplished with the strategies:

furlong → mi → km

and

s → min → h

The template for the calculation is

$$\frac{9}{108}\,\frac{furlongs}{s} = \frac{9}{108}\,\frac{furlongs}{s}\left(\frac{mi}{furlong}\right)\left(\frac{km}{mi}\right)\left(\frac{s}{\text{min}}\right)\left(\frac{\text{min}}{h}\right)$$

Putting in the numbers gives

$$\frac{9}{108}\frac{furlong}{s} = \frac{9}{108}\frac{\cancel{furlong}}{\cancel{s}}\left(\frac{1\ \cancel{mi}}{8\ \cancel{furlong}}\right)\left(\frac{1.609344\ km}{1\ \cancel{mi}}\right)\left(\frac{60\ \cancel{s}}{1\ \cancel{min}}\right)\left(\frac{60\ \cancel{min}}{1\ h}\right)$$

$$= \frac{(9)(1.609344)(60)(60)}{(108)(8)}\frac{km}{h} = 60.3504\ \frac{km}{h}$$

Thus, this horse would be averaging a speed of 60.35 km/h, rounded to two decimal places, over the duration of this race.

Example 8: A rectangular region of forest measuring 5.6 mi by 9.2 mi was destroyed by fire. Compute the area of this region and express your final answer in units of hectares.

solution:
Just applying the formula for the area of a rectangle gives

A = LW = (5.6 mi)(9.2 mi) = 51.52 mi^2

as the area of the region. What we must do now is convert this area to units of hectares. We have that

1 hectare = 10 000 m^2
1 mi = 5280 ft

and

1 ft = 30.48 cm

and we know how to convert centimetres to metres. Thus a promising strategy to use here is

mi $\rightarrow$ ft $\rightarrow$ cm $\rightarrow$ m

each repeated twice (because we have two units of 'mi' to begin with, which must eventually be converted to give m^2 in total) and then

$m^2 \rightarrow$ hectare (ha)

The required template of calculations is

$$51.52\ mi^2 = 51.52\ mi^2\left(\frac{ft}{mi}\right)^2\left(\frac{cm}{ft}\right)^2\left(\frac{m}{cm}\right)^2\left(\frac{ha}{m^2}\right)$$

With numbers inserted, this gives

$$51.52\ mi^2 = 51.52\ mi^2\left(\frac{5280\ ft}{1\ mi}\right)^2\left(\frac{30.48\ cm}{1\ ft}\right)^2\left(\frac{1\ m}{100\ cm}\right)^2\left(\frac{1\ ha}{10000\ m^2}\right)$$

$$= \frac{(51.52)(5280)^2(30.48)^2}{(100)^2(10000)}\ ha = 13343.6187\ ha$$

Rounding to the nearest whole number, the area of the burned region is 13344 hectares.

<u>Example 9</u>: The acceleration due to gravity near the earth's surface (denoted conventionally by the symbol g) is 9.81 m/s^2. This means that the speed of a falling object (in the absence of air resistance) will increase by 9.81 m/s for each second of fall. Convert this value into units of ft/s^2, and units of km/h/s.

solution:
The two calculations requested here are relatively independent. First, to carry out the conversion from m/s^2 to ft/s^2, we note that the time units stay the unchanged, and so we need simply to convert the length units from metres to feet. This is easily done using the strategy

m → cm → ft

The template for the calculation is

$$9.81\frac{m}{s^2} = 9.81\frac{m}{s^2}\left(\frac{\quad cm}{\quad m}\right)\left(\frac{\quad ft}{\quad cm}\right)$$

With the numbers in, the final answer obtained is

$$9.81\frac{m}{s^2} = 9.81\frac{\cancel{m}}{s^2}\left(\frac{100\ \cancel{cm}}{1\ \cancel{m}}\right)\left(\frac{1\,ft}{30.48\ \cancel{cm}}\right)$$

$$= \frac{(9.81)(100)}{(30.48)}\frac{ft}{s^2} = 32.185\ \frac{ft}{s^2}$$

Since the value of g we were given has three significant figures, we should probably round our final answer here to three significant figures, stating it as 32.2 ft/s^2.

The second conversion requested is a bit more complicated. The length unit must be converted from metres to kilometres, which is easy by itself. Also, one of the time units must be converted from seconds to hours. Thus, the following strategies are required:

m → km and s → min → h

The required template for the calculation is

$$9.81\frac{m}{s^2} = 9.81\frac{m}{s^2}\left(\frac{\quad km}{\quad m}\right)\left(\frac{\quad s}{\quad \min}\right)\left(\frac{\quad \min}{\quad h}\right)$$

Putting the numbers in, we get

$$9.81\frac{m}{s^2} = 9.81\frac{\cancel{m}}{\cancel{s^2}^{\,s}}\left(\frac{1\,km}{1000\,\cancel{m}}\right)\left(\frac{60\,\cancel{s}}{1\,\cancel{min}}\right)\left(\frac{60\,\cancel{min}}{1\,h}\right)$$

$$= \frac{(9.81)(60)(60)}{1000}\frac{km}{s\bullet h} = 35.316\,\frac{km}{s\bullet h}$$

Rounding to three significant figures again, we state the final answer as 35.3 km/h/s. The units written this way indicate the meaning of this number a bit more clearly. When an object falls without air resistance near the surface of the earth, its speed will increase by 35.3 km/h for each second that it falls.

Example 10: Typical barometric pressure as reported in Canada is 101 kPa = 101,000 N/m^2. Convert this value to its equivalent in pounds per square inch, the units used for barometric pressure in the United States.

solution:
In our table of unit conversion factors, we have

1 N = 0.22480894 lb

and

1 in = 2.54 cm

and we know how to convert from centimetres to metres. Thus, an effective strategy here is the pair of transformations

$N \rightarrow lb$ and $m \rightarrow cm \rightarrow in$

The second path in the line above must be executed twice, because the length units are squared in the initial quantity. The template required for this conversion is

$$101\,000\,\frac{N}{m^2} = 101\,000\,\frac{N}{m^2}\left(\frac{\quad lb}{\quad N}\right)\left(\frac{\quad m}{\quad cm}\right)^2\left(\frac{\quad cm}{\quad in}\right)^2$$

Now, insert the numbers and do the arithmetic:

$$101\,000\,\frac{N}{m^2} = 101\,000\,\frac{\cancel{N}}{\cancel{m^2}}\left(\frac{0.22480894\,lb}{1\,\cancel{N}}\right)\left(\frac{1\,\cancel{m}}{100\,\cancel{cm}}\right)^2\left(\frac{2.54\,\cancel{cm}}{1\,in}\right)^2$$

$$= \frac{(101\,000)(0.22480894)(2.54)^2}{(100)^2}\frac{lb}{in^2} = 14.64881131\,\frac{lb}{in^2}$$

Rounding off to three significant figures, we get the final answer of 14.6 lb/in^2.

<u>Example 11</u>: It is estimated that approximately 1.05 mi^3 of rock was blasted into the atmosphere as dust during the major eruption of Mt. St. Helens in May, 1980. To how many truckloads of 12 m^3 each does this volume of rock correspond?

solution:
Obviously if we knew the volume of rock in units of m^3, this would be quite an easy problem to solve. Thus, we start by converting 1.05 mi^3 to its equivalent in cubic metres.

We know that

1 mi = 5280 ft

and

1 ft = 30.48 cm

Thus, an appropriate strategy for carrying out this unit conversion is

mi → ft → cm → m

This will have to be repeated three times, since three units of miles must be converted to three units of metres. The required template for the calculation is

$$1.05\ mi^3 = 1.05\ mi^3 \left(\frac{ft}{mi}\right)^3 \left(\frac{cm}{ft}\right)^3 \left(\frac{m}{cm}\right)^3$$

Putting the numbers in, we get

$$1.05\ mi^3 = 1.05\ \cancel{mi^3} \left(\frac{5280\ \cancel{ft}}{1\ \cancel{mi}}\right)^3 \left(\frac{30.48\ \cancel{cm}}{1\ \cancel{ft}}\right)^3 \left(\frac{1\ m}{100\ \cancel{cm}}\right)^3$$

$$= \frac{(1.05)(5280)^3(30.48)^3}{(100)^3}\ m^3 = 4{,}376{,}590{,}917\ m^3$$

This number is so big that we've broken the SI rule against using commas in numbers in writing it out. To get the number of truckloads, we just have to divide this number by 12 (which gives 364,715,909.7) and then round up to the nearest whole truckload. Thus, the rock blown into the atmosphere as dust during the eruption was equivalent to 364,715,910 truckloads of 12 m^3 each. (Actually, since the original estimate of the volume of this rock was given to just three significant figures, we should probably round our final answer off to three significant figures as well, stating it as 365,000,000 truckloads – that 365 million truckloads!)

<u>Example 12</u>: Certain types of beverages (not "Kool-Aid") are typically sold in bottles containing a volume called a "fifth," which is actually one-fifth of a U. S. liquid gallon (or four-fifths of a U. S. liquid quart). Determine how many millilitres of beverage this volumes equals.

solution:
This problem can be solved quite quickly. We have the unit conversion factor

1 U. S. gal (liq) = 3.785306 litres

in our table of unit conversion factors. Thus, there is a very simple strategy here:

U. S. gal (liq) → litre → millilitre

Abbreviating 'U. S. gal' as simply 'gal' here, the required template for this calculation is

$$\frac{1}{5}\,gal = \frac{1}{5}\,gal\left(\frac{L}{gal}\right)\left(\frac{ml}{L}\right)$$

Putting in the numbers then gives

$$\frac{1}{5}\,gal = \frac{1}{5}\,\cancel{gal}\left(\frac{3.785306\,\cancel{L}}{1\,\cancel{gal}}\right)\left(\frac{1000\,ml}{1\,\cancel{L}}\right)$$

$$= \left(\frac{1}{5}\right)(3.785306)(1000)\,ml = 757.0612\,ml$$

Thus, a "fifth" of, uh, beverage, is exactly 757.0612 ml. (This probably explains why certain types of beverage commonly come in containers of 750 ml.)

Example 13: Astronomers measure certain distances in units of au or 'astronomical units,' which are defined to be 92,955,807 miles – the average distance between the centers of the earth and the sun. They define a second unit called the parsec, which is equal to 206,264.4 au. (There is a reason for this value being chosen, but we don't want to get into too many technicalities here.) Now, a parsec is a pretty fair distance, but when we get to talking about intergalactic space, it's more convenient to use units of kiloparsecs. The question is this: how many kiloparsecs is there in a zettametre?

solution:
If your first answer was "who cares?" you are not exhibiting the right attitude of adventure and challenge this problem represents. This is quite a complicated question, and it deals with units of measurement that few of us use in everyday conversation. Nevertheless, the unit conversion method we've been illustrating throughout this document is up to the task of solving even this problem.

Start by summarizing the information we have so far:

1 au = 92,955,807 mi
1 parsec = 206264.4 au
1 kiloparsec = 1000 parsec

and

1 zettametre = 1 Zm = 10^{21} metre

Now, we need a strategy to convert from Zm to kiloparsecs. A bit of thought suggests the following:

Zm → m → km → mi → au → parsec → kiloparsec

The template required here is now rather easy to write out, even if it is also rather lengthy:

$$1\ \text{Zm} = 1\ \text{Zm}\left(\frac{\text{m}}{\text{Zm}}\right)\left(\frac{\text{km}}{\text{m}}\right)\left(\frac{\text{mi}}{\text{km}}\right)\left(\frac{\text{au}}{\text{mi}}\right)\left(\frac{\text{parsec}}{\text{au}}\right)\left(\frac{\text{kparsec}}{\text{parsec}}\right)$$

Putting the numbers in gives:

$$1\ \text{Zm} = 1\ \text{Zm}\left(\frac{10^{21}\ \text{m}}{1\,\text{Zm}}\right)\left(\frac{1\,\text{km}}{1000\ \text{m}}\right)\left(\frac{1\,\text{mi}}{1.609344\ \text{km}}\right)\left(\frac{1\,\text{au}}{92{,}955{,}807\ \text{mi}}\right)$$

$$\times\left(\frac{1\,\text{parsec}}{206264.4\ \text{au}}\right)\left(\frac{1\,\text{kparsec}}{1000\ \text{parsec}}\right)$$

$$= \frac{\left(10^{21}\right)}{(1000)(1.609344)(92{,}955{,}807)(206264.4)(1000)}\ \text{kiloparsecs}$$

$$= 32.40786\ \text{kiloparsecs}$$

Thus, 1 Zm is equivalent to 32.41 kiloparsecs, rounded to two decimal places. Even though you probably have no mental image of how big several of these units of length really are, you should be able to set up the calculations to complete the conversion and be able to confirm that no errors have been made in the calculation.