Reading: “Relevant Costs and Revenues” (ECON 4550 Coursepak, Page 1)

Time Value of Money, Future Value, and Present Value:

(Q): Is “$1,000 today” more valuable than “$1,000 one year from now”?
(A): Yes. But why?
- One possible “use” of “$1,000 today”: Take “$1,000 today” and do nothing with it (i.e., put it in an envelope in your bedroom closet) and one year from now you have “$1,000 one year from now” => “$1,000 today” cannot be worth less than “$1,000 one year from now”
- A second possible “use” of $1,000 today”: Take “$1,000 today” and invest it in an interest bearing savings account (e.g., 1% guaranteed return from ING Direct) and one year from now you have “more than $1,000 one year from now” (e.g., ($1,000)(1.01) = ($1,010) => “$1,010 one year from now”)

Time Value of Money – the recognition that, so long as a positive rate of return can be earned, any particular amount of money is worth more the sooner it is received

- The notions of Present Value and Future Value follow from this recognition
What will be the “future value” (“t periods from now”) of receiving “$X today,” supposing that we could earn a return of “r percent” in each period?

When we stash away “$X today,” then…
- one period from now we have: $X(1+r)
- two periods from now we have: $[X(1+r)](1+r)
- three periods from now we have: $([X(1+r)](1+r))(1+r)$, which is simply equal to $X(1+r)^3$
- …t periods from now we have: $X(1+r)^t$

That is, $FV(X) = X(1+r)^t$
- this tells us the “value in t periods of stashing away $X today”

From here: $FV(X) = X(1+r)^t$

$\Leftrightarrow Y = X(1+r)^t$

$\Leftrightarrow X = \frac{Y}{(1+r)^t}$

$\Leftrightarrow PV(Y) = \frac{Y}{(1+r)^t}$

That is, $PV(Y) = \frac{Y}{(1+r)^t}$
- this tells us the “amount of money we would have to stash away today in order to have exactly $Y by period t”

From here, we can calculate the “net present value” of different streams of investments (i.e., future payments)
Example:

- **Alternative 1** – spend $1,000 today in order to receive $500 in each of the next four years
- **Alternative 2** – spend $1,000 today in order to receive $420 in each of the next five years
- Suppose the relevant discount rate is 8%
- Suppose there is a capital constraint of $1,000 (so that it is not possible to make both investments)

(?) Which of the two investments is the better choice?

The Net Present Value of Alternative 1 is:

\[
NPV_1 = -1,000 + \frac{500}{(1.08)} + \frac{500}{(1.08)^2} + \frac{500}{(1.08)^3} + \frac{500}{(1.08)^4}
\]

\[
\approx -1,000 + 462.96 + 428.67 + 396.92 + 367.51
\]

\[
= 656.06
\]

The Net Present Value of Alternative 2 is:

\[
NPV_2 = -1,000 + \frac{420}{(1.08)} + \frac{420}{(1.08)^2} + \frac{420}{(1.08)^3} + \frac{420}{(1.08)^4} + \frac{420}{(1.08)^5}
\]

\[
\approx -1,000 + 388.89 + 360.08 + 333.41 + 308.71 + 285.84
\]

\[
= 676.93
\]

“Alternative 2” is a better choice than “Alternative 1,” since the “Net Present Value” is greater => choose “Alternative 2”
In the discussion thus far, the money to be received/paid at each point in time has been (by assumption) “certain,” as opposed to “uncertain”

- In practice this is clearly not realistic for many decisions that firms must make => more often than not, decision making involves “uncertainty” or “risk”

Let $p_i$ denote the “probability of ‘event i’ occurring”

- for each possible ‘event i’ we must have $0 \leq p_i \leq 1$
- further, if there are a total of ‘N different possible events,’ then $\sum_{i=1}^{N} p_i = 1$

**Example:**

- Suppose I toss “two fair coins” and pay you $1,000 if “both come up heads” and $13,000 if “at least one comes up tails”
- That is: $p_{HH} = .25, V_{HH} = 1,000$ and $p_T = .75, V_T = 13,000$

**Certainty Equivalent** – the certainty equivalent (of a gamble, G) is the unique amount of money for which an individual is indifferent between taking the gamble (G) versus receiving the money for certain

- for the gamble described above, determine the largest amount of money (that would be received for certain) for which you would still prefer to take the gamble
For \( p_{HH} = .25 \), \( V_{HH} = 1,000 \) and \( p_T = .75 \), \( V_T = 13,000 \), would you rather have the gamble or:

- $1,000 for certain?
  - clearly the gamble is better…
- $13,000 for certain?
  - clearly the certain payment is better…

So, we have quickly narrowed the search for the value of the Certainty Equivalent down to the range of somewhere between $1,000 and $13,000. But, how can we go further?

**Expected Value** – the expected value of a random variable is equal to the weighted average of all possible realized values of the random variable

- In general, for a random variable \( V \) that can take on “\( N \) different possible values,” each with “probability of \( 0 \leq p_i \leq 1 \),” the Expected Value is: \( E(V) = \sum_{i=1}^{N} p_i V_i \)
- For the gamble describe above, the Expected Value is: \( E(G) = (.25)(1000) + (.75)(13000) = 10,000 \)

**Expected Value Criterion** – the Expected Value Criterion for decision-making in the face of risk dictates that the decision-maker should choose the available option with the greatest expected value
“Expected Value Criterion” may be reasonable for some decision-makers (e.g., perhaps a large, multi-national corporation), but is not the criterion used by individuals, households, or many enterprises.

“Expected Value Criterion” is only “correct” if the decision-maker is “risk neutral” => but most individuals and many enterprises are “risk averse.”

Would you rather have the gamble or $10,000 for certain?
- personally, I’d prefer $10,000 for certain… => seems reasonable to expect most people to prefer “$10,000 for certain” => Certainty Equivalent should be somewhere between $1,000 and $10,000
- personally, I’d likely prefer $9,000 for certain…
- is someone with a Certainty Equivalent of less than $10,000 irrational?

**Utility of Wealth:**

- Suppose that an individual’s preferences for wealth can be summarized by the Bernoulli Utility Function $u(x)$
- In general, for a random variable (V) that can take on “N different possible values,” each with “probability of $0 \leq p_i \leq 1$,” the value of von Neumann-Morgenstern Expected Utility is: $U(V) = \sum_{i=1}^{N} p_i u(V_i)$

Note, this differs from the Expected Value, $E(V) = \sum_{i=1}^{N} p_i V_i$, so long as $u(v) \neq v$

For the gamble being considered above:
$U(G) = (.25)u($1,000$) + (.75)u($13,000$)$
**Expected Utility Criterion** – the Expected Utility Criterion for decision-making in the face of risk dictates that the decision-maker should choose the available option with the greatest value of von Neumann-Morgenstern Expected Utility.

Are there any reasonable expectations regarding the shape or properties of the function \( u(x) \)?

1. **“More is better”** (or “monotonicity” or “positive marginal utility”) – utility should become larger as the amount of wealth a person has is increased => \( MU(x) = u'(x) > 0 \)

2. **“Diminishing Marginal Utility of Wealth”** – the marginal utility of wealth (while always positive) should likely become smaller as the amount of wealth a person has is increased => \( MU'(x) = u''(x) < 0 \)

To understand these properties, consider the following scenario:

- Suppose you go to an Atlanta Falcons game with a $100 bill…
  - If you give the $100 to Arthur Blank, how will you have changed his utility?
  - If you instead give the $100 to a homeless person on the sidewalk outside of the Georgia Dome, how will you have changed his utility?

  - Seems reasonable that
    - either person would be made better off by the additional $100 (\( MU(x) = u'(x) > 0 \) for each)
    - the increase in well-being would be larger for the homeless person than for Arthur Blank (suggesting that \( MU'(x) = u''(x) < 0 \))
Example: consider \( u(x) = 2\sqrt{x} \)

- \( MU(x) = u'(x) = 2\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}} > 0 \) so “monotonicity” is satisfied
- \( MU'(x) = u''(x) = -\left(\frac{1}{2}\right)x^{-\frac{3}{2}} = -\frac{1}{2x^{\frac{3}{2}}} < 0 \) so the person has a “diminishing marginal utility of wealth”
Returning to the gamble of \( p_{HH} = .25, \ V_{HH} = 1,000 \) and \( p_T = .75, \ V_T = 13,000, \) what is the value of the “certainty equivalent” to this gamble for someone with \( u(x) = 2\sqrt{x} \)?

- To answer this question we need to find a “dollar amount \((C)\)” for which \( u(C) \) is exactly equal to \( U(G) \) (i.e., the expected utility of the gamble)

- Recall, we already noted that \( U(G) \) is equal to \( (.25)u(1,000) + (.75)u(13,000) \)

- Thus, the certainty equivalent must satisfy:
  \[
  u(C) = (.25)u(1,000) + (.75)u(13,000)
  \]
  \[
  \Leftrightarrow 2\sqrt{C} = (.25)(2)\sqrt{1,000} + (.75)(2)\sqrt{13,000}
  \]
  \[
  \Leftrightarrow \sqrt{C} = (.25)\sqrt{1,000} + (.75)\sqrt{13,000}
  \]
  \[
  \Leftrightarrow C = \left( (.25)\sqrt{1,000} + (.75)\sqrt{13,000}\right)^2
  \]
  \[
  \Leftrightarrow C \approx 8,727.08
  \]

- Note, as anticipated, the Certainty Equivalent is “somewhere between $1,000 and $10,000”
**Different Types of Risk Preferences:**

**Risk Aversion** – an individual is risk averse if for any gamble \( G \), the Certainty Equivalent of \( G \) is less than the expected value of \( G \)

- Risk Averse \( \iff MU'(x) = u''(x) < 0 \iff \text{“diminishing marginal utility of wealth”} \)

**Risk Neutrality** – an individual is risk neutral if for any gamble \( G \), the Certainty Equivalent of \( G \) is exactly equal to the expected value of \( G \)

- Risk Neutral \( \iff MU'(x) = u''(x) = 0 \iff u(x) = Ax \) with \( A > 0 \iff \text{“constant marginal utility of wealth”} \)

**Risk Seeking** – an individual is risk seeking if for any gamble \( G \), the Certainty Equivalent of \( G \) is greater than the expected value of \( G \)

- Risk Seeking \( \iff MU'(x) = u''(x) > 0 \iff \text{“increasing marginal utility of wealth”} \)

Again, for most individuals, in most contexts “diminishing marginal utility of wealth” or “risk aversion” is what we would expect.
Updating Probabilities => Bayes’ Rule

- Suppose:
  - You have some “information” or “beliefs” about the world
  - You obtain some “new information”
  - How should you “combine” the “new information” with your initial information” in order to “update your beliefs”?

- **Bayes’ Rule**: discussed in further detail on Pages 77-80 of Coursepak (within the discussion of “Canonical Decision Problems”)

- General formula for Bayes Rule:
  \[ P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{\sum_j P(A_j)P(B \mid A_j)} \]

Notation and Terminology:

- The original information is called **prior** information and it conveys different probabilities of different events occurring (each denoted \( P(i) \))

- Combining these prior probabilities with new information generates **posterior** probabilities that are conditional on the information obtained (denoted \( P(i \mid "Information") \))

- Our notational convention will be to enclose any obtained information in quotation marks, in order to indicate that it may or may not in fact be accurate
The Quality of Information:
- The quality of information can be assessed by examining the source from which it comes
- Typically this assessment is made based upon previous observations and interactions with the provider of the information

Example of updating beliefs via Bayes’ Rule…
- Suppose your company is contemplating the development and introduction of a new product
- Demand can either be high or low
- Based on past experience your initial estimates of the probabilities of these two possible outcomes are: \( P(h) = \frac{3}{4} \) and \( P(l) = \frac{1}{4} \)
- Suppose you hire a marketing firm to conduct some focus groups and surveys with potential customers in order to gauge future demand => will report either “High” or “Low”
  - clearly, the information provided by the marketing firm is not always “100% correct”
- Based upon past experiences, suppose that you believe
  - \( P("H"| h) = \frac{9}{10} \) (i.e., if demand is truly high, the marketing firm will tell you “High” 9 out of 10 times)
  - \( P("L"| h) = \frac{1}{10} \) (i.e., if demand is truly high, the marketing firm will tell you “Low” 1 out of 10 times)
  - \( P("L"| l) = \frac{4}{5} \) (i.e., if demand is truly low, the marketing firm will tell you “Low” 4 out of 5 times)
  - \( P("H"| l) = \frac{1}{5} \) (i.e., if demand is truly low, the marketing firm will tell you “High” 1 out of 5 times)
After you get the report from the marketing firm, exactly what should you believe about the probabilities associated with future demand?

- For example, if they report “High,” how likely is it that demand is actually high? => that is, what is $P(h \mid "H")$?
- Similarly, what are $P(l \mid "H")$, $P(l \mid "L")$, and $P(h \mid "L")$?

By Bayes’ Rule:

$$P(h \mid "H") = \frac{P(h)P("H\mid h)}{P(h)P("H\mid h) + P(l)P("H\mid l)} = \frac{\left(\frac{3}{4}\right)\left(\frac{9}{10}\right)}{\left(\frac{3}{4}\right)\left(\frac{9}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{5}\right)} = \frac{27}{29} \approx .9310$$

$$P(l \mid "H") = \frac{P(l)P("H\mid l)}{P(l)P("H\mid l) + P(h)P("H\mid h)} = \frac{\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)}{\left(\frac{1}{4}\right)\left(\frac{1}{5}\right) + \left(\frac{3}{4}\right)\left(\frac{9}{10}\right)} = \frac{2}{29} \approx .0690$$

$$P(l \mid "L") = \frac{P(l)P("L\mid l)}{P(l)P("L\mid l) + P(h)P("L\mid h)} = \frac{\left(\frac{1}{4}\right)\left(\frac{4}{5}\right)}{\left(\frac{1}{4}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{10}\right)} = \frac{8}{11} \approx .7273$$

$$P(h \mid "L") = \frac{P(h)P("L\mid h)}{P(h)P("L\mid h) + P(l)P("L\mid l)} = \frac{\left(\frac{3}{4}\right)\left(\frac{1}{10}\right)}{\left(\frac{3}{4}\right)\left(\frac{1}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{4}{5}\right)} = \frac{3}{11} \approx .2727$$

Let’s draw a diagram to help us see why these formulas and values are correct...
Note: all the probabilities stated above were given…
To see how to derive the posterior probabilities by applying Bayes’ Rule, conceptually think about this scenario playing itself out 1,000 times…
So, if you are told “High,” what should you think?

- There are a total of $675+50=725$ instances in which you would be told “High”
  - That is, $P(h)P("H"| h) + P(l)P("H"| l)$
- Demand is actually high in 675 of these instances
  - That is, $P(h)P("H"| h)$
- Thus, after being told “High,” you reasonably update your belief that demand is actually high to a probability of
  \[
P(h|"H") = \frac{675}{675+50} = \frac{675}{725} \approx 0.9310
\]

Similarly,

- $P(l|"H") = \frac{50}{675+50} = \frac{50}{725} \approx 0.0690$
- $P(l|"L") = \frac{200}{200+75} = \frac{200}{275} \approx 0.7273$
- $P(h|"L") = \frac{75}{200+75} = \frac{75}{275} \approx 0.2727$
We could illustrate these posterior probabilities as follows:

\[
P(h|"H") \approx 0.9310 \\
P(l|"H") \approx 0.0690 \\
P(h|"L") \approx 0.2727 \\
P(l|"L") \approx 0.7273
\]

- What we have done here is referred to as “flipping the tree”
- Recognize that the “conditionals” are flipped…
  - We started with *probabilities over true demand* and *beliefs about information conditional on demand*
  - We ended up with *probabilities over information* and *beliefs about true demand conditional on information*
- We will ultimately use this “flipped tree” to determine whether or not it pays to acquire information at a cost
Reflection on “Quality of Information”…

Recognize, the “quality of information” is dictated by the values of the conditional probabilities \( P("H"|h) \), \( P("L"|h) \), \( P("L"|l) \), and \( P("H"|l) \)

- Generally, it would seem as if having any one of these values “closer to 1” would be an indication of “higher quality information”

(?) Does having \( P("H"|h) = 1 \) imply that “perfect information” is being revealed?
- What if \( P("H"|h) = 1 \) and \( P("H"|l) = 1 \) ?

(?) In order to convey “perfect information” do we need \( P("H"|h) = 1 \) and \( P("L"|l) = 1 \) ?
- What if \( P("H"|h) = 0 \) and \( P("L"|l) = 0 \) ? [see question on problem set]
**Relevant Costs and Revenues:**

When making decisions it is critical to focus on the relevant costs and relevant benefits of different courses of action…

- ECON 2100 tells us that what matters most for decision making is Marginal Revenue and Marginal Costs
- If a decision maker focuses on the “incorrect” costs or benefits, then poor choices can easily be made

**Common decision making error:** incorrectly allocating Fixed (or Quasi-Fixed) Costs

**Example: “A Tale of Three Products”** (Page 2 of Coursepak)

<table>
<thead>
<tr>
<th></th>
<th>Product A</th>
<th>Product B</th>
<th>Product C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Per-Unit Revenue</strong></td>
<td>$10</td>
<td>$15</td>
<td>$5</td>
</tr>
<tr>
<td><strong>Per-Unit Variable Cost</strong></td>
<td>$5</td>
<td>$5</td>
<td>$3</td>
</tr>
<tr>
<td><strong>Monthly Quantity Sold</strong></td>
<td>10,000</td>
<td>2,000</td>
<td>2,500</td>
</tr>
<tr>
<td><strong>Contribution</strong></td>
<td>$50,000</td>
<td>$20,000</td>
<td>$5,000</td>
</tr>
</tbody>
</table>

**Contribution** – the difference between total revenue and total variable costs for a particular product-line

- Essentially what we previously defined as Producer’s Surplus (i.e., Profit plus Fixed Costs of production), but for one particular product of several products that the firm is producing
Suppose the firm must incur an overhead of $60,000 if they produce any of the three product lines (but can entirely avoid the overhead if they instead shutdown ⇒ it’s a “Quasi-Fixed Cost”)

- Which product lines should the firm produce?

Seems reasonable to allocate the $60,000 overhead across all product lines that are ultimately produced…

- If the overhead is allocated equally across each of the three lines, then there is an overhead of $20,000 for each line
  - From here it looks as if “Product C” is not worth producing ⇒ $5,000 Contribution does not cover $20,000 overhead
  - Drop “Product C”
- If the overhead is allocated equally across each of the two remaining lines, then there is an overhead of $30,000 for each line
  - Now it looks as if “Product B” is not worth producing ⇒ $20,000 Contribution does not cover $30,000 overhead
  - Drop “Product B”
- Now the entire overhead must be allocated to “Product A”
  - It now appears as if “Product A” is not worth producing ⇒ $50,000 Contribution does not cover $60,000 overhead
  - Drop “Product A”
- So, by this logic the firm will not produce any of the three product lines

But, the firm can earn a positive profit of:

($50,000)+($20,000)+($5,000)−($60,000) = ($15,000)

by producing all three lines…
The previous logic was flawed in that it arbitrarily assigned fixed costs in such a way so as to make them appear “variable”

- Doing so clouded the firms decision making, so that in the end they easily could have made a poor decision

Again, in practice it is critical to properly recognize how different decisions do or do not alter the relevant costs and benefits of the firm

- In a complex real world this is not as straightforward as it is in a classroom/textbook discussion of economics

Especially easy to make poor decisions when a firm

(i) Produces more than one product, especially if there are substitution and/or complementarity effects between the goods
- For Toyota a *Yaris* is (to some degree) a substitute for a *Corolla*
- For Kodak a “new model of printer” is a substitute for a “previous model of printer,” and an ink cartridge is a complement to each type of printer

(ii) Resources have more than one potential use (Opportunity Costs)
- Perhaps producing a particular product requires the use of some warehouse space, but if the product is not produced the warehouse space could be sublet to a third party => when determining the costs of producing the product, be sure to include the forgone rent as a cost of choosing to produce the product