Definitions and Concepts:

- **Decision Analysis** – a logical and systematic approach for analyzing decision-making problems

- Four distinct steps of decision-making:
  1. Decision Structuring
  2. Assessment and Information Gathering
  3. Evaluation of the Decision Problem
  4. Sensitivity Analysis

- **Decision Structuring** consists of “clearly defining the problem,” by
  1a. identifying the set of alternatives or available options from which the decision-maker is able to choose
  1b. recognizing any uncertainties (i.e., factors beyond the control of the decision-maker) that affect the outcome
  1c. determining the criteria (e.g., “maximizing expected profit”) for choosing among the available alternatives

- **Assessment and Information Gathering** consists of
  2a. determining the likelihood of or probability associated with any uncertainties within the decision problem
  2b. calculating an appropriate value for each possible outcome that could result from the decision process

- **Evaluation of the Decision Problem** consists of the careful consideration and analysis of the information and structure described in the first two steps, in order to determine the best course of action

- **Sensitivity Analysis** consists of analyzing the problem in more general terms, in order to determine the degree to which the optimal choice is dependent upon changes in probabilities, valuations of outcomes, or other assumptions made when formulating the decision problem

- Two basic elements of a decision tree: decision nodes and chance nodes

- **Decision nodes** indicate a choice to be made by the decision-maker (depicted by a “solid circle”)

Reading: “Decision Analysis” (ECON 4550 Coursepak, Page 53)
- **Chance nodes** indicate an outcome to be determined by someone other than the decision-maker and are viewed by the decision maker as the result of “nature” or “chance” (depicted by an “unshaded circle”)

- **Common “Decision Traps”:**
  1. **Specifying the problem incorrectly** => during the phase of “decision structuring,” sufficient attention must be given to specifying the “correct problem”
     - Make sure all relevant available courses of action are considered
     - Make sure the correct valuations are placed on different outcomes
  2. **Overconfidence when specifying probabilities** => managers are often not particularly good at estimating probabilities (and, in practice, are typically wildly overconfident in their judgments); try not to be led astray by such systematic biases
  3. **Bias toward recent information/experience** => the best assessments of probabilities should be based upon all available relevant information
     - when accounting for all available information, many managers tend to place “too much weight” on recent experience (and thus “too little weight” on historical information)
     - care should be given to make sure you are not too easily influenced by recent information/experiences
Example of a simple decision problem:

Evaluation of the decision problem (“Step 3” of the decision-making process)…
- The choices described in the decision tree are best analyzed by “Folding Back the Tree” => “Backward Induction”
- A good manager is “forward looking, but reasons backward”

(?) If she chooses to “Continue R&D” and the outcome of the process is “Failure,” should she still go ahead and “Make Proposal”?

What would be the best course of action within this portion of the decision tree?

That is, what would be the best course of action at this decision node?
**Pruning the Tree...**

Starting at the endpoints of the tree or “terminal nodes,”

- replace each chance node with the corresponding “expected value” of the relevant decision
  - within the portion of the tree under examination, the expected profit of choosing to “Make Proposal” is:
    \[
    (.05)($600,000) + (.95)(–$250,000) \\
    ($30,000) + (–$237,500) \\
    = $(-207,500)
    \]

- compare the expected payoffs of the different courses of action and eliminate the less desirable option
  - if she instead chooses “Don’t Make Proposal,” then she gets a payoff of $(-200,000) for certain => at this point the better choice is “Don’t Make Proposal”

- continue to “Prune the Tree” via “Backward Induction”

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<table>
<thead>
<tr>
<th>Continue R&amp;D</th>
<th>Make Proposal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Win</td>
<td>Make Proposal</td>
</tr>
<tr>
<td>($600,000)</td>
<td>(.90)</td>
</tr>
<tr>
<td>Lose</td>
<td>(.10)</td>
</tr>
<tr>
<td>(-250,000)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Abandon R&amp;D</th>
<th>Don't Make Proposal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Win</td>
<td>$0</td>
</tr>
<tr>
<td>Lose</td>
<td>($-200,000)</td>
</tr>
</tbody>
</table>

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By reasoning similar to what we did above,

- If she chooses to “Continue R&D” and the outcome of the process is “Success,” she should choose “Make Proposal” (Expected Payoff of $515,000 is greater than certain loss of $(-200,000)…)

- For the initial decision, the expected payoff of “Continue R&D” is (assuming optimal behavior at later nodes):
  \[
  (.4)[(.9)($600,000)+ (.1)(–$250,000)] + (.6)(–$200,000) \\
  = (.4)($515,000) + (.6)(–$200,000) \\
  = $206,000 – $120,000 = $86,000
  \]

  - Since this is greater than $0 (i.e., the payoff from initially choosing “Abandon R&D”), the better initial choice is “Continue R&D”

Her best choices are (note, we need to specify a choice for each and every decision node):

1. Start by choosing to “Continue R&D”
2. If the R&D is a “Success,” then “Make Proposal”
3. If the R&D is a “Failure,” then “Don’t Make Proposal”
Example of Conducting Sensitivity Analysis:

- It is often insightful to conduct a “Sensitivity Analysis” to determine the degree to which the optimal choices depend upon the values of probabilities, valuations of outcomes, or other assumptions made when formulating the problem (“Step 4” of the decision-making process).

1. Let the “Probability that Continued R&D is a ‘Success’” be denoted by \( p \) => for what range of \( p \) would she still want to choose “Continue R&D” initially?

The expected payoff from choosing “Continue R&D” is now:

\[
(p)\left[ (.9)(600,000) + (.1)(-250,000) \right] + (1-p)(-200,000)
\]

\[
= (715p-200)(1,000)
\]

Choose “Continue R&D” initially if and only if this positive:

\[
715p > 200 \Rightarrow p > \frac{200}{715} \approx .2797
\]
2. Let the “Probability that she wins the contract from a proposal following Continued R&D that is a ‘success’” be denoted by \( q \) [suppose the previously altered probability is again \( .4 \), and not \( p \)]

For what range of \( q \) would she still want to choose “Make Proposal” following “Continued R&D” that is a “success”?
- This is still the better choice if and only if:

\[
(q)(600,000) + (1 - q)(-250,000) \geq (-200,000)
\]
\[
\Rightarrow 600,000q - 250,000 + 250,000q \geq -200,000
\]
\[
\Rightarrow 850,000q \geq 50,000
\]
\[
\Rightarrow q \geq \frac{5}{35} = \frac{1}{7} \approx .058824
\]

Assuming \( q \geq .058824 \), for what range of \( Q \) would she still want to choose “Continued R&D” initially?
- This is still the better choice if and only if:

\[
.4[(q)(600,000) + (1 - q)(-250,000)] + .6[(-200,000)] \geq 0
\]
\[
\Rightarrow .4[850,000q - 250,000] - .6[200,000] \geq 0
\]
\[
\Rightarrow 340,000q - 220,000 \geq 0
\]
\[
\Rightarrow q \geq \frac{220}{340} = \frac{11}{17} \approx .647059
\]
Multiple Choice Questions:

1. _____________ is defined as a logical and systematic approach for analyzing decision-making problems.
   A. Decision Analysis
   B. Regression Analysis
   C. Expected Utility Maximization
   D. Game Theory

2. Which of the following is NOT one of the four distinct steps of decision-making?
   A. Decision Structuring.
   B. Determining the value of the Certainty Equivalent.
   C. Assessment and Information Gathering.
   D. None of the above answers is correct (since each choice is one of the distinct steps of decision-making).

3. ________________ consists of analyzing a decision problem in more general terms, in order to determine the degree to which the optimal choice is dependent upon changes in probabilities, valuations of outcomes, or other assumptions made when formulating the decision problem.
   A. Decision Structuring
   B. Profit Maximization
   C. Risk Avoidance
   D. Sensitivity Analysis

4. Which of the following is one of the common “decision traps” identified and discussed in lecture?
   A. Specifying the problem incorrectly.
   B. Overconfidence when specifying probabilities.
   C. Complete disregard for recent information/experience.
   D. More than one (perhaps all) of the above answers is correct.

5. Two basic elements of a decision tree are “decision nodes” and “chance nodes.” At a “decision node,” a choice is made by
   A. “nature.”
   B. the person formulating the decision problem.
   C. some person other than the individual formulating the decision problem.
   D. None of the above answers are correct.
Problem Solving or Short Answer Questions:

1. Consider the decision problem of a firm described by the decision tree below:

   - **1A.** Suppose the firm initially chooses “Continue.” Following this choice, if “chance” chooses “Failure,” would the firm want to choose “Enter” or Don’t Enter”? Explain.
   - **1B.** Suppose the firm initially chooses “Continue.” Following this choice, if “chance” chooses “Success,” would the firm want to choose “Enter” or Don’t Enter”? Explain.
   - **1C.** Should the firm choose “Continue” or “Abandon” initially? Explain.

2. Consider a firm that is contemplating the development of a new product. In order to develop the new product, they must incur development costs of $175,000 (these costs can be entirely avoided if they do not develop the new product). Demand for the new product may either be “high,” “medium,” or “low.” From the perspective of the firm, demand is uncertain and will not be observed until after the product is developed and brought to market. The probability of, quantity sold, per unit price, and costs of production under each market condition is summarized in the table below:

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
<th>Quantity Sold</th>
<th>Per Unit Price</th>
<th>Costs of Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>.20</td>
<td>100,000</td>
<td>$9.00</td>
<td>$500,000</td>
</tr>
<tr>
<td>Medium</td>
<td>.65</td>
<td>75,000</td>
<td>$8.00</td>
<td>$400,000</td>
</tr>
<tr>
<td>Low</td>
<td>.15</td>
<td>60,000</td>
<td>$7.00</td>
<td>$330,000</td>
</tr>
</tbody>
</table>

   - **2A.** Draw a decision tree which summarizes the decision problem of this firm.
   - **2B.** Should the firm develop this new product or not? Explain.
3. Consider a firm selling a product with inverse demand of \( P_d(q) = 20 - (0.005)q \). The firm currently has production costs of \( C(q) = 10q + 2,000 \). They have the option of attempting to develop a new technology that would lower production costs to \( \hat{C}(q) = 4q + 2,000 \). Research and development costs are $3,315 and (if undertaken) must be incurred regardless of whether or not the new technology is “successful” or a “failure.” If the firm attempts to develop the new technology, their innovation will be successful with probability \( p = \frac{20}{29} \). Throughout your analysis, restrict attention to the profit/loss of the firm in only the current period (i.e., assume that the firm will not be operating in any future period) and assume that the firm is risk neutral (i.e., that the firm simply wants to maximize expected profit).

3A. Draw a decision tree which summarizes the decision problem of this firm.
3B. Should the firm undertake Research and Development efforts in order to attempt development of this new production technology? Explain.
3C. Suppose instead that research and development costs are $4,095. Should the firm undertake Research and Development efforts in order to attempt development of this new production technology? Explain.
3D. Suppose instead that research and development costs are $4,095 and that innovation will be a success with probability \( p = \frac{23}{40} \) (and will be a failure with probability \( 1 - p = \frac{17}{40} \)). Should the firm undertake Research and Development efforts? Explain.

4. Heidi manages a firm that is going to start operating in a new market. She has the option of targeting either “high end” consumers or “low end” consumers. She has some uncertainty regarding actual market conditions in each of these two segments. Very loosely, she thinks that market conditions in each segment could be either “favorable” or “unfavorable.” The probability of “favorable” versus “unfavorable” conditions in each market segment (along with her resulting profit in each case) is:

<table>
<thead>
<tr>
<th>Market</th>
<th>Condition</th>
<th>Probability</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>High end</td>
<td>Favorable</td>
<td>( p )</td>
<td>$800,000</td>
</tr>
<tr>
<td></td>
<td>Unfavorable</td>
<td>1–( p )</td>
<td>$200,000</td>
</tr>
<tr>
<td>Low end</td>
<td>Favorable</td>
<td>( q )</td>
<td>$500,000</td>
</tr>
<tr>
<td></td>
<td>Unfavorable</td>
<td>1–( q )</td>
<td>$400,000</td>
</tr>
</tbody>
</table>

4A. Draw the decision tree for this decision problem.
4B. If \( p = \frac{2}{5} \) and \( q = \frac{3}{4} \), which segment should she serve? Explain.
4C. Suppose \( p = \frac{2}{5} \). For what range of \( q \) should she serve the “low end” consumers?
4D. Suppose \( q = \frac{3}{4} \). For what range of \( p \) should she serve the “low end” consumers?
4E. Charlie claims, “If \( p > \frac{1}{3} \), then you should serve the ‘high end’ consumers (regardless of the value of \( q \)). If \( p < \frac{1}{3} \), then you should serve the ‘low end’ consumers (regardless of the value of \( q \)).” Derive a general condition, in terms of both \( p \) and \( q \), that Heidi can use to determine which market segment to serve. From this condition, verify that Charlie’s advice is indeed correct.
Answers to Multiple Choice Questions:

1. A  
2. B  
3. D  
4. D  
5. B

Answers to Problem Solving or Short Answer Questions:

1A. At this point, choosing “Don’t Enter” gives the firm a certain payoff of $(–125,000). Choosing “Enter” would instead give the firm an expected payoff of:

\[(.20)(\$185,000) + (.80)(\$[-200,000]) = \$37,000 + \$(–160,000) = \$(–123,000).\]

Since this figure is less negative than the payoff from choosing “Don’t Enter,” the better choice of the firm at this point is to “Enter.”

1B. At this point, choosing “Don’t Enter” gives the firm a certain payoff of $(200,000). Choosing “Enter” would instead give the firm an expected payoff of:

\[(.75)(\$400,000) + (.25)(\$120,000) = \$300,000 + \$30,000 = \$330,000.\]

Since this figure is greater than the payoff from choosing “Don’t Enter,” the better choice of the firm at this point is to “Enter.”

1C. Given the answers to parts (1A) and (1B), if the firm initially chooses “Continue,” then they will realize an expected payoff of \$330,000 with probability (.70) and an expected payoff of $(–123,000) with probability (.30). This yields an expected payoff from choosing “Continue” of:

\[(.70)(\$330,000) + (.30)(\$[-123,000]) = \$231,000 + \$(–36,900) = \$194,100.\]

Choosing “Abandon” gives an expected payoff of:

\[(.05)(\$240,000) + (.95)(\$20,000) = \$12,000 + \$19,000 = \$31,000.\]

Thus, the better choice initially is “Continue.”

2A. The decision tree for this problem is:

```
      High
     /     \
   (.20)  (.65)
Develop  Medium
     /     \      /     \
   (.15)  (.15)  (.20)  (.10)
Don’t Develop  Low  Develop  Don’t Develop

\$250,000 = \$900,000 – \$500,000 – \$175,000
\$25,000 = \$600,000 – \$400,000 – \$175,000
\$(–85,000) = \$420,000 – \$330,000 – \$175,000
\$0
```
2B. From the decision tree illustrated above, if the firm chooses “Don’t Develop,” then they realize a certain payoff of $0. If instead they choose “Develop,” then they realize an expected payoff of:

\[
(0.20)(225,000) + (0.65)(25,000) + (0.15)(-85,000) = 45,000 + 16,250 + (-12,750) = 48,500.
\]

Thus, the better choice for the firm is to “Develop” the new product.

3A. Start by recognizing that this firm has Marginal Revenue of \( MR(q) = 20 - (0.01)q \). If they must operate using the current technology (for which \( MC(q) = 10 \)), then the will sell 1,000 units of output (this quantity is determined by setting \( MR(q) = MC(q) \) \( \iff \) \( 20 - (0.01)q = 10 \) and solving for \( q \)) and charge a price of $15 per unit (this price is determined by evaluating \( P_D(q) = 20 - (0.005)q \) at the optimal quantity of 1,000 units). This gives the firm a profit (not accounting for any research and development costs) of \( \pi = (15 - 10)(1,000) - 2,000 = 3,000 \).

If they instead operate with the improved technology (for which \( MC(q) = 4 \)), then the will sell 1,600 units of output (this quantity is determined by setting \( MR(q) = MC(q) \) \( \iff \) \( 20 - (0.01)q = 4 \) and solving for \( q \)) and charge a price of $12 per unit (this price is determined by evaluating \( P_D(q) = 20 - (0.005)q \) at the optimal quantity of 1,600 units). This gives the firm a profit (not accounting for any research and development costs) of \( \pi = (12 - 4)(1,600) - 2,000 = 10,800 \).

Denoting Research and Development Costs by \( R \), the decision tree can be drawn as:

\[
\begin{align*}
\text{Successful} & \quad p = 0.45 \quad \$10,800 - R = \$7,485 \\
\text{Develop} & \\
\text{Don’t Develop} & \quad 1-p = 0.55 \quad \$3,000 - R = \$(-315) \\
&
\end{align*}
\]

3B. From the decision tree drawn above, we see that if the firm chooses “Don’t Develop,” then they will earn a certain payoff of $3,000. If instead they choose “Develop,” then their expected payoff is:

\[
(0.45)(7,485) + (0.55)(-315) = 3,195
\]

Thus, the better choice is to attempt to “Develop” the new production technology.

3C. If instead Research and Development Costs were $4,095, then the expected payoff of choosing “Develop” to start would be:

\[
(0.45)(6,705) + (0.55)(-1,095) = 2,415
\]

Thus, the better choice is “Don’t Develop” the new production technology.

3D. With this different value for the probability of “Successful” innovation, the expected payoff of choosing “Develop” to start is:

\[
(0.575)(6,705) + (0.425)(-1,095) = 3,390
\]

Thus, the better choice is to attempt to “Develop” the new production technology.
4A. Based upon the given information, the decision tree in this situation is:

```
Target High
   Favorable
     p
   1-p
   Unfavorable
   $800,000

Target Low
   Favorable
     q
   1-q
   Unfavorable
   $500,000
   $400,000
```

4B. If \( p = \frac{2}{5} \) and \( q = \frac{1}{4} \), then her expected payoff from targeting the “high end” customers is:

\[
(.4)(800,000) + (.6)(200,000) = $440,000
\]

and her expected payoff from targeting the “low end” customers is:

\[
(.75)(500,000) + (.25)(400,000) = $475,000.
\]

Thus, in this case she should choose to target the “low end” customers.

4C. As noted, for \( p = \frac{2}{5} \) her expected payoff from serving “high end” customers is $440,000. As a function of \( q \), her expected payoff from serving “low end” customers is:

\[
q(500,000) + (1-q)(400,000) = 100,000q + 400,000
\]

Thus, her expected payoff is greater from targeting “low end” consumers if and only if:

\[
100,000q + 400,000 > 440,000 \iff 100,000q > 40,000 \iff q > \frac{40}{100} = .4
\]

4D. As noted, for \( q = \frac{1}{4} \) her expected payoff from serving “low end” customers is $475,000. As a function of \( p \), her expected payoff from serving “high end” customers is:

\[
p(800,000) + (1-p)(200,000) = 600,000p + 200,000
\]

Thus, her expected payoff is greater from targeting “low end” consumers if and only if:

\[
475,000 > 600,000p + 200,000 \iff 275,000 > 600,000p \iff \frac{275}{600} = .458\overline{3} > p
\]

4E. As noted, as a function of \( q \), her expected payoff from targeting “low end” customers is:

\[
q(500,000) + (1-q)(400,000) = 100,000q + 400,000
\]

and as a function of \( p \), her expected payoff from targeting “high end” customers is:

\[
p(800,000) + (1-p)(200,000) = 600,000p + 200,000
\]

Thus, targeting “low end” customers is better if and only if:

\[
100,000q + 400,000 > 600,000p + 200,000 \iff q + 2 > 6p \iff p + \frac{1}{2} < q + \frac{1}{3}
\]

Note that the right side of the inequality is increasing in \( q \), while the left side is increasing in \( p \). For the largest possible value of \( q \) (i.e., \( q = 1 \)) this condition is \( p < \frac{1}{2} \). Thus, Charlie’s claim that, “If \( p > \frac{1}{2} \), then you should serve the ‘high end’ consumers (regardless of the value of \( q \))” is correct. Further, for the smallest possible value of \( q \) (i.e., \( q = 0 \)) this condition is \( p < \frac{1}{3} \). Thus, Charlie’s claim that, “If \( p < \frac{1}{3} \), then you should serve the ‘low end’ consumers (regardless of the value of \( q \))” is also correct.