Definitions and Concepts:

- Within this topic we are going to analyze four different scenarios in which the previously developed basic framework of “Decision Analysis” can be applied
  1. “Litigation Decision” (Page 70 of Coursepak)
  2. “Principal-Agent Problem”
  3. “Product Development Sequencing” (Page 67 of Coursepak)
  4. “Market Research” (Pages 69-70 of Coursepak)

- **Probability of Two Independent Events** – the probability of two independent events both occurring is simply equal to the product of the probabilities of each individual event occurring

- **Principal-Agent Problems** – difficulties that arise with getting an agent (e.g., employee) to behave in a way that is desired by a principal (e.g., owner of a firm)
  - such problems are fundamentally caused by differences in incentives between the principal and the agent
  - often exacerbated by incomplete or asymmetric information
The decision tree illustrated above could be simplified to:

From here, recognize that the expected value of choosing “don’t settle” is:

\[
(0.6)(-63,000) + (0.3)(-13,000) + (0.1)(-3,000) = -42,000
\]

- That is, an expected cost of $42,000
- Thus, if her goal is to simply minimize her expected payment, then she should be willing to settle out of court initially if and only if \( S \leq 42,000 \)
What if she is instead “risk averse”?

- The largest amount of $S$ that she would be willing to pay up-front in order to settle out of court should now be greater than $42,000 \implies$ that is, she’d be willing to pay a premium in order to avoid the risk of going to trial
- If we make assumptions on her preferences for wealth (and on her initial amount of wealth), then we could again determine an exact value for the most she’d be willing to pay up-front
- Assume her preferences are summarized by $u(x) = \sqrt{x}$ and that she initially has wealth of $W = 75,000$
- Under these assumptions the decision tree becomes:

- Thus, her Expected Utility from choosing “don’t settle” is:
  \[ U(ds) \approx (.6)(109.545) + (.3)(248.998) + (.1)(268.328) = 167.2592 \]
- It follows that she would want to settle up-front if and only if:
  \[ u(75,000 - S) \geq U(ds) \]
  \[ \iff \sqrt{75,000 - S} \geq 167.2592 \]
  \[ \iff 75,000 - S \geq 27,975.64 \]
  \[ \iff S \leq 47,024.46 \]
- As expected, when she is risk averse the amount she is willing to settle for up-front is greater than it was when she was risk neutral ($47,024.46$, as opposed to $42,000$)
“Principal-Agent Problem”: [building upon “R&D Example” from previous topic]

- Recall our previous example of Sarah Chang’s decision over continued R&D and subsequent proposal for the development of an electronic timing system…

- We noted that the that would maximize expected profit were:
  1. Start by choosing to “Continue R&D”
  2. If the R&D is a “Success,” then “Make Proposal”
  3. If the R&D is a “Failure,” then “Don’t Make Proposal”

- The firm owners have agreed to pay her a bonus of 1% of the annual profits of her division
- Year-to-date profit thus far is $300,000, and this is the final potential new project of the year
- Suppose she is risk averse with preferences summarized by \( u(x) = \sqrt{x} \) (recall, it is often reasonable to expect large firms to be risk neutral but individuals to be risk averse) and wants to maximize her expected utility
- Will this lead to any difference in her behavior?

\[
\begin{align*}
\text{Continue R&D} & \quad \text{Win} \quad \text{Lose} \\
\text{Make Proposal} & \quad 90,000 \quad (10) \\
\text{Success} & \quad 500 \quad (10) \\
\text{Abandon R&D} & \quad 3,000 \quad 54.77
\end{align*}
\]
If she chooses “Continue R&D” and it is a “Failure,” she will still choose “Don’t Make Proposal”:
\[(.05)\sqrt{9,000} + (.95)\sqrt{500} \approx 25.99 < 31.62 \approx \sqrt{1,000}\]

If she chooses “Continue R&D” and it is a “Success,” she will still choose “Make Proposal”:
\[(.90)\sqrt{9,000} + (.10)\sqrt{500} \approx 87.62 > 31.62 \approx \sqrt{1,000}\]

But now, the expected payoff from choosing “Continue R&D” is less than that from choosing “Abandon”:
\[(.36)\sqrt{9,000} + (.04)\sqrt{500} + (.60)\sqrt{1,000} \approx 54.02 < 54.77 \approx \sqrt{3,000}\]

She will choose “Abandon” initially, even though the firm owners (who desire to maximize expected profit) would want her to choose “Continue R&D.”

When an Agent (Sarah) and Principal (firm owners) have different objectives/preferences, the agent might not choose to take the actions that are in the best interest of the principal. Sarah chose to “Abandon” the project, even though “Continue R&D” was the choice that the owners would have preferred.
**“Product Development Sequencing”:** [Page 67 of Coursepak]

Three critical questions:
1. What different options are available in terms of the timing of the different phases of research?
2. Suppose the firm attempts to make the board in-house. How should they proceed if the goal is to minimize expected costs?
3. Should the firm purchase the circuit boards outside at the start or attempt to make the board in-house?

1. They could possibly:
   a. first pursue research on Development and, if successful, then proceed to research on Fabrication
   b. first pursue research on Fabrication and, if successful, then proceed to research on Development
   c. simultaneously pursue research on both Development and Fabrication
   d. not pursue any research (if they do not attempt to make the board in-house)

The “entire decision tree” (in terms of possible outcomes) is:
Recognize, if they choose…

- “d”:
  - they do not need to incur any research costs
  - they are certain to be able to buy circuit boards from an outside vendor for $24 each

- “c”:
  - they are certain to incur research costs of $80,000
  - if either avenue is unsuccessful, they still have enough time to buy circuit boards from an outside vendor for $24 each

- “a” or “b”:
  - they are certain to incur some research costs
  - if the first research efforts are unsuccessful, then they can still have enough time to buy circuit boards from an outside vendor for $24 each
  - if the first research efforts are successful but the second research efforts are not, then they have to pay a total price of $34 each to obtain circuit boards from an outside vendor

2. Of options “a,” “b,” or “c,” which is best?

- Option “a” => Development first
  - The expected costs of Option “a” are:
    \[(.72)(170,000) + (.18)(420,000) + (.10)(290,000) = 227,000\]

- Option “b” => Fabrication first
  - Thus, the expected costs of Option “b” are:
    \[(.72)(170,000) + (.08)(420,000) + (.20)(270,000) = 210,000\]
Option “c” => Development & Fabrication simultaneously

- Success (.72)
- Failure (.28)

<table>
<thead>
<tr>
<th>Success</th>
<th>$170,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure</td>
<td>$320,000</td>
</tr>
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- since we need both to be a “success” in order for “overall success,” the probability of “success” is (.8)(.9) = (.72) and the probability of “failure” is 1 − (.8)(.9) = (.28)
- Thus, the expected costs of Option “c” are:
  
  \[
  (.72)(170,000) + (.28)(320,000) = 212,000
  \]

- Between “a,” “b,” and “c,” the best choice (i.e., the one with lowest expected cost) is Option “b” => Fabrication first (followed by Development, if Fabrication is a success)

3. Should they attempt to make the board in-house or should they simply purchase the board from an outsider at the start? (i.e., is Option “d” better than Option “b”?)
- Option “d” results in certain costs of:
  
  \[
  (24)(10,000) = 240,000
  \]
- Thus, Option “b” is the overall best choice (i.e., they should attempt to make the board in-house, by first pursuing Fabrication
Three initial questions:
1. If you knew actual demand, under which realizations of demand would you want to introduce the new formula?
2. Without knowing actual demand, would you introduce the new formula?
3. Imagine a market research firm that provides perfect information (i.e., if the firm says that annual sales will be “x units,” then annual sales will in fact be “x units” for certain). What is the maximum price you would be willing to pay for the services of such a firm?

1. The per unit contribution is simply: $(12)–(7)=(5)$.  
   - high => total contribution is $(5)(600,000) = 3$ million. Since this is greater than $2.1$ million, the new formula should be introduced.
   - mid => total contribution is $(5)(300,000) = 1.5$ million. Since this is less than $2.1$ million, the new formula should not be introduced.
   - low => total contribution is $(5)(200,000) = 1$ million. Since this is less than $2.1$ million, the new formula should not be introduced.

2. Without knowing actual demand, we face the following decision:

![Decision Tree Image]

The expected contribution from introducing the new formula is:

$$[12 – 7][.3)(600,000) + (.4)(300,000) + (.3)(200,000)] = 1,800,000$$

Since this is less than certain payoff of $2,100,000 from maintaining the old formula, expected profit is maximized by NOT introducing the new formula.
3. If given the option of hiring this “perfect marketing firm,” then we would face the following decision:

Applying our insights from “Question 1,” we can easily see that the expected contribution after choosing Hire is:

\[ (0.3)(3.0) + (0.4)(2.1) + (0.3)(2.1) \]

Thus, would be willing to pay up to \((2.37) - (2.1) = (0.27) = 270,000\) to hire a market research firm that provides “perfect information”

- **Corollary:** since “less than perfect information” is “less valuable,” the maximum willingness to pay to hire a research firm that provides “less than perfect information” will be strictly less than \(270,000\).

**Marble Research** – You are considering hiring Marble Research, one of the nation’s leading market research companies. For \(150,000\) they will perform a test market study on your new formula. Although Marble can’t predict sales figures exactly, they will indicate whether they believe the product will be a “Hit” or a “Flop.”

Your beliefs about their report are: \(P(H|h) = 1, \ P(F|h) = 0, \ P(H|m) = \frac{3}{8}, \ P(F|m) = \frac{5}{8}, \ P(H|l) = \frac{1}{6}, \) and \(P(F|l) = \frac{5}{6} \).

**Four additional questions:**
4. What are the values of \(P(h|H), \ P(m|H), \ P(l|H), \ P(h|F), \ P(m|F), \) and \(P(l|F)\)?
5. Should you introduce the new Silkskin formula if Marble Research indicates “Hit”?
6. Should you introduce the new Silkskin formula if Marble Research indicates “Flop”?
7. Would you be willing to hire Marble Research at their asking price of \(150,000\)? What is the maximum amount you would be willing to pay for their information?
4. Applying Bayes’ Rule:

Thus:

\[
P(H|H) = \frac{\frac{3}{10} \cdot 3}{\frac{4}{10} \cdot \frac{3}{8} + \frac{3}{10} \cdot 3 + \frac{3}{10} \cdot 0} = \frac{6}{33} = \frac{2}{11}
\]

\[
P(H|F) = \frac{\frac{3}{10} \cdot 3}{\frac{4}{10} \cdot \frac{3}{8} + \frac{4}{10} \cdot \frac{5}{8} + \frac{3}{10} \cdot 0} = \frac{18}{55}
\]

\[
P(F|H) = \frac{\frac{3}{10} \cdot 0}{\frac{4}{10} \cdot \frac{3}{8} + \frac{3}{10} \cdot 3 + \frac{3}{10} \cdot 0} = \frac{0}{21} = 0
\]

\[
P(F|F) = \frac{\frac{3}{10} \cdot 0}{\frac{4}{10} \cdot \frac{3}{8} + \frac{4}{10} \cdot \frac{5}{8} + \frac{3}{10} \cdot 0} = \frac{0}{55} = 0
\]

5. If Marble Research reports “Hit,” then you face the following decision:

Thus, your expected profit from choosing to introduce the New formula is:

\[
[(.6)(3.0) + (.3)(1.5) + (.1)(1.0)] = 2.35
\]

Since $2.35 million is greater than $2.1 million, if you hire Marble Research and they come back with a report of “Hit,” then expected contribution is maximized by introducing the New formula.
6. If Marble Research reports “Flop,” then you face the following decision:

Thus, your expected profit from choosing to introduce the New formula is:

\[
25.1 \times 0.1 + 5 \times 0.3 + 0 = 2.225 \text{ million}
\]

Since $1.25 million is less than $2.1 million, if you hire Marble Research and they come back with a report of “Flop,” then expected contribution is maximized by NOT introducing the New formula.

7. What is our “maximum willingness to pay” to hire Marble Research?
   - We have determined that if they report “Hit,” we will choose to introduce the New formula and realize an expected contribution of $2.35 million.
   - Also, if they report “Flop,” we will choose to NOT introduce the New formula and realize an expected contribution of $2.10 million.
   - From our answer to Question 4, recognize:
     \[
     P("H") = \frac{6}{20} + \frac{3}{20} + \frac{1}{20} = \frac{10}{20} = 0.5
     \]
     \[
     P("F") = 0 + \frac{5}{20} + \frac{5}{20} = \frac{10}{20} = 0.5
     \]
   - Thus, if we hire Marble Research, the expected contribution is:
     \[
     0.5 \times 2.35 + 0.5 \times 2.10 = 2.225
     \]
     That is, $2.225 million = 2,225,000.
   - Since we would realize a certain payoff of $2.1 million from not hiring Marble Research (and not introducing the New formula), the most we should be willing to pay to hire Marble Research is:
     \[
     (2.225 \text{ million}) - (2.1 \text{ million}) = \frac{125 \text{ million}}{125,000}
     \]
   - So, no, we would not be willing to hire them for $150,000.
   - Finally, note that (as expected) this amount is less than our “maximum willingness to pay” of $270,000 for the marketing firm that would provide “perfect information”.
Bargain Research Consortium – The Bargain Research Consortium (BRC), one of Marble’s competitors, is offering you a similar market study for just $15,000. They will similarly indicate whether they expect the product will be a “Hit” or a “Flop.”

Your beliefs about their report are: $P("H"| h) = \frac{3}{5}$, $P("F"| h) = \frac{2}{5}$, $P("H"| m) = \frac{2}{5}$, $P("F"| m) = \frac{3}{5}$, $P("H"| l) = \frac{1}{5}$, and $P("F"| l) = \frac{4}{5}$.

Four final questions:
8. What are the values of $P(h | "H")$, $P(m | "H")$, $P(l | "H")$, $P(h | "F")$, $P(m | "F")$, and $P(l | "F")$?
9. Should you introduce the new Silkskin formula if BRC indicates “Hit”?
10. Should you introduce the new Silkskin formula if BRC indicates “Flop”?
11. Would you be willing to hire BRC at their asking price of $15,000? What is the maximum amount you would be willing to pay for their information?

8. Applying Bayes’ Rule:

Thus:

$P(h | "H") = \frac{\frac{9}{20} + \frac{9}{20} + \frac{3}{20}}{\frac{3}{20} + \frac{3}{20} + \frac{1}{20}} = \frac{9}{20} = (.45)$, $P(m | "H") = \frac{\frac{8}{20} + \frac{8}{20} + \frac{5}{20}}{\frac{3}{20} + \frac{3}{20} + \frac{1}{20}} = \frac{8}{20} = (.4)$, $P(l | "H") = \frac{\frac{3}{20} + \frac{3}{20} + \frac{4}{20}}{\frac{3}{20} + \frac{3}{20} + \frac{1}{20}} = \frac{3}{20} = (.15)$

$P(h | "F") = \frac{\frac{6}{30} + \frac{12}{30} + \frac{12}{30}}{\frac{6}{30} + \frac{12}{30} + \frac{12}{30}} = \frac{6}{30} = (.2)$, $P(m | "F") = \frac{\frac{12}{30} + \frac{12}{30} + \frac{12}{30}}{\frac{6}{30} + \frac{12}{30} + \frac{12}{30}} = \frac{12}{30} = (.4)$, and $P(l | "F") = \frac{\frac{12}{30} + \frac{12}{30} + \frac{12}{30}}{\frac{6}{30} + \frac{12}{30} + \frac{12}{30}} = \frac{12}{30} = (.4)$
9. If BRC reports “Hit,” then you face the following decision:

Thus, your expected profit from choosing to introduce the New formula is:

\[ \left[ (.45)(3.0) + (.4)(1.5) + (.15)(1.0) \right] = 2.1 \]

Since $2.1 million is exactly equal to $2.1 million, if you hire BRC and they come back with a report of “Hit,” then your expected contribution is exactly the same from either introducing or NOT introducing the New formula. (i.e., you are exactly indifferent between the two options.)

10. If BRC reports “Flop,” then you face the following decision:

Thus, your expected profit from choosing to introduce the New formula is:

\[ \left[ (.2)(3.0) + (.4)(1.5) + (.4)(1.0) \right] = 1.6 \]

Since $1.6 million is less than $2.1 million, if you hire BRC and they come back with a report of “Flop,” then expected contribution is maximized by NOT introducing the New formula.

11. What is our “maximum willingness to pay” to hire BRC?

- We have determined that if they report “Hit,” we will realize an expected contribution of $2.10 million (by either choosing to introduce or not introduce the New formula)
- Also, if they report “Flop,” we will choose to NOT introduce the New formula and realize an expected contribution of $2.10 million
- Thus, the expected contribution after hiring BRC is exactly $2.1 million, which can be realized by NOT hiring them and simply choosing to not introduce the New formula
- That is, hiring them adds nothing to our expected contribution! => we should not be willing to pay them anything for their information!
- So, no, we would not be willing to hire them for $15,000
**Multiple Choice Questions:**

1. Difficulties that arise with getting an employee to take actions that are in the best interest of her employer are described as
   A. Decision Analysis.
   B. the Litigation Decision.
   C. a Principal-Agent Problem.
   D. Price Discrimination.

2. Tom is going to randomly draw one ball from “Urn A” and one ball from “Urn B.” “Urn A” is filled with 20 white balls and 80 black balls. “Urn B” is filled with 30 red balls and 70 green balls. What is the probability that Tom will draw “a white ball from ‘Urn A’ and a green ball from ‘Urn B’”?
   A. (.14).
   B. (.25).
   C. (.70).
   D. (.90).

For Questions 3 and 4, consider the following scenario. Dorsey went on a vacation to the Bahamas. He asked James to come over to his apartment to feed and check up on his cat while he was away. After driving to Dorsey’s apartment, James realized he did not have money for the parking meter outside of the building. If he parks on the street without putting money in the meter, he risks having to pay a $145 fine. James expects that he would get a ticket with probability $p$. Alternatively, he can use his debit card to pay $4 to park in a parking lot right next to the building. Suppose that his initial level of wealth is $629 and that his preferences for risk are characterized by the Bernoulli Utility Function $u(x) = 4\sqrt{x}$.

3. If James chooses to park in the parking lot, he will realize utility of
   A. 88 with probability $p$ and 100 with probability $1 - p$.
   B. 88 for certain.
   C. 100 for certain.
   D. 484 with probability $p$ and 629 with probability $1 - p$.

4. For which of the following values of $p$ should James choose to park illegally on the street (i.e., risk getting parking ticket)?
   A. $p = .0125$.
   B. $p = .05$.
   C. he should park illegally for both of the above values of $p$.
   D. he should not park illegally for either of the above values of $p$.
Problem Solving or Short Answer Questions:

1. Last year Lawrence opened a new business, a coffee house called “Latte-Larry’s.” During his first month of business, a 19 year old-woman named Hollie bought a cup of coffee at “Latte-Larry’s” which she subsequently spilled while driving home (resulting in second degree burns on her legs). She has brought a lawsuit against Lawrence, claiming that his coffee was too hot and that the lid he provides for customers is inadequate.

Hollie’s lawyer has indicated that she would be willing to settle out of court for a payment of $80,000. Lawrence must decide whether he should accept this offer or let the case proceed. If the case moves forward, the first step would be for the judge to make a summary judgment, essentially deciding if her case has any merit whatsoever. It will cost Lawrence $4,000 in legal fees to have his attorney prepare for the summary judgment. Following the summary judgment, the judge will decide to hear the case with probability \( p \) and instead dismiss the case with probability \( 1 - p \). If the case goes to trial, Lawrence will have to incur an additional $12,000 in legal fees. He expects that he would lose the case and have to pay an expected award of $100,000 with probability \( q \) (and that he would win the case and have to make no payment with probability \( 1 - q \)).

Assume throughout that Lawrence has initial wealth of $125,000 and that he is risk averse, with preferences that can be summarized by \( u(x) = \ln(\frac{x}{1,000}) \). (Recall: (i) if \( \ln(z) = y \), then \( z = \exp\{y\} = e^y \) and (ii) \( \ln(z) - \ln(y) = \ln(\frac{z}{y}) \).)

1A. Draw the full decision tree (with two chance nodes) that Lawrence faces.
1B. Draw a simplified decision tree (with only one chance node) that equivalently describes Lawrence’s decision problem.
1C. Supposing \( p = .8 \) and \( q = .75 \), should Lawrence settle out of court or let the case proceed? Explain.
1D. Continuing to suppose \( p = .8 \), determine the range of \( q \) for which he should choose to settle out of court.
1E. Again supposing \( q = .75 \), determine the range of \( p \) for which he should choose to settle out of court.
1F. Derive a general condition in terms of \( p \) and \( q \) that specifies when he should choose to settle out of court.
You are the manager of the New Product Development Division at Peachtree Beverages, a soft drink company based in Atlanta, GA. You are contemplating bringing a new pecan flavored cola to market, but are uncertain of consumer demand. You presently expect that demand will be either High or Low, with respective probabilities of (.25) and (.75). In order to bring the product to market, you must incur up-front costs of $400,000. If demand is high, this product would generate a contribution of $1,200,000; if demand is low, this product would generate a contribution of $300,000. Assume throughout that your goal is to simply maximize expected profit.

You are contemplating hiring one of two marketing firms (either “The Veblen Group” or “Kwality Research”) to give you a better idea of consumer demand for this product. If you hire either firm, you will be given a report of either “Hit” or “Flop.” “The Veblen Group” charges $75,000 for their services, while “Kwality Research” charges only $2,000.

Your beliefs about the information to be provided by “The Veblen Group” can be summarized by the following conditional probabilities: 

\[ P_V(H|H) = 0.9, \]
\[ P_V(F|H) = 0.1, \]
\[ P_V(H|L) = 0.05, \]
\[ P_V(F|L) = 0.95. \]

Similarly, your beliefs about the information to be provided by “Kwality Research” can be summarized by the following conditional probabilities: 

\[ P_K(H|H) = 0.8, \]
\[ P_K(F|H) = 0.2, \]
\[ P_K(H|L) = 0.4, \]
\[ P_K(F|L) = 0.6. \]

You have four possible courses of action: (i) don’t bring the new product to market, (ii) bring the new product to market without hiring either firm, (iii) hire “The Veblen Group” and (after receiving their report) then decide whether or not to bring the product to market, or (iv) hire “Kwality Research” and (after receiving their report) then decide whether or not to bring the product to market.

2A. If you hire “The Veblen Group” and they report “Hit,” would you choose to bring the product to market? If you hire “The Veblen Group” and they report “Flop,” would you choose to bring the product to market? Explain.

2B. Based upon your answers to part (2A), what is your expected payoff from hiring “The Veblen Group”?

2C. If you hire “Kwality Research” and they report “Hit,” would you choose to bring the product to market? If you hire “Kwality Research” and they report “Flop,” would you choose to bring the product to market? Explain.

2D. Based upon your answers to part (2C), what is your expected payoff from hiring “Kwality Research”?

2E. How should you proceed: (i) don’t bring the new product to market, (ii) bring the new product to market without hiring either firm, (iii) hire “The Veblen Group” and (after receiving their report) then decide whether or not to bring the product to market, or (iv) hire “Kwality Research” and (after receiving their report) then decide whether or not to bring the product to market? Explain.
3. Teresa is the manager of “Iris Cellular,” a provider of communications services currently operating in North Carolina and Virginia. Teresa must decide whether her company should start providing services in Tennessee – she is faced with a choice of either entering or not entering this new market. If she chooses to not enter this new market, company profits for the year will be $640,000 for certain. If “Iris Cellular” starts providing service in Tennessee, their entry into this new market will be either a “success” (with probability \( p \)) or a “failure” (with probability \( 1 - p \)). If it is a “success,” total profits will be $800,000 greater than otherwise; if it is a “failure,” total profits will be $280,000 smaller than otherwise. The owners of “Iris Cellular” have a goal of maximizing expected profit.

With the intention of aligning Teresa’s incentive with their own, the owners of the firm have included a performance bonus in Teresa’s contract. In addition to her annual salary, she is paid a bonus of 1% of company profits. Suppose that her preferences over this bonus are summarized by the Bernoulli utility function \( u(x) = \sqrt{x} \).

3A. Draw the decision tree that Teresa faces.
3B. For what values of \( p \) would Teresa choose to “enter” the new market? Explain.
3C. For what values of \( p \) would the firm owners want Teresa to choose to “enter” the new market? Explain.
3D. Based upon your answers above, specify a value of \( p \) for which Teresa would NOT make the decision that the firm owners prefer.

4. Yawges Inc. is attempting to develop and bring to market a new personal transportation device. This project is currently dubbed “Project MaryAnn.” There are two remaining phases of Research and Development that must be undertaken successfully in order to bring this product to market: “Electrical Engineering R&D” and “Mechanical Engineering R&D.” The two phases of R&D use different sets of engineers and can proceed independently. Each phase takes five months. Undertaking “Electrical Engineering R&D” costs $120,000 and will be “successful” with probability (.85). Undertaking “Mechanical Engineering R&D” costs $180,000 and will be “successful” with probability (.95). Additionally, the firm must incur an overhead cost of \( x = 7,500 \) per month during each month that any R&D is taking place. If Yawges is able to successfully bring “Project MaryAnn” to market, they will be able to realize revenue of $1,000,000. If Yawges goes forward with R&D, they essentially have three options: (i) “EE R&D” first, followed by “ME R&D” if “EE R&D” is a “success,” (ii) “ME R&D” first, followed by “EE R&D” if “ME R&D” is a “success,” and (iii) “EE R&D” and “ME R&D” simultaneously. Answer the following questions under an assumption that the firm wants to maximize expected profit.

4A. If Yawges were to move forward with R&D sequentially, would they be better off pursuing “ME R&D” first or “EE R&D” first? Explain.
4B. Is Yawges better off pursuing R&D sequentially or simultaneously? Explain.
4C. Would your answers to questions (4A) and (4B) differ if the monthly overhead costs had instead been only \( x = 5,500 \)? Explain.
Answers to Multiple Choice Questions:

1. C  
2. A  
3. C  
4. A

Answers to Problem Solving or Short Answer Questions:

1A. Draw the full decision tree (with two chance nodes) that Lawrence faces:

1B. Draw a simplified decision tree (with only one chance node) that equivalently describes Lawrence’s decision problem:

1C. Supposing $p = .8$ and $q = .75$, Lawrence’s expected utility from choosing “don’t settle” is:

\[
.2 \ln(121) + .2 \ln(109) + .6 \ln(9) \approx 3.21576
\]

which is less than his certain utility of $\ln(45) \approx 3.80666$ which can be realized by choosing to settle. Thus, for these probabilities he should choose to settle out of court.
1D. Continuing to suppose \( p = .8 \), Lawrence’s expected utility (as a function of \( q \)) from choosing “don’t settle” can be expressed as:

\[
(2 \ln(121) + (.8)(1 - q) \ln(109) + (.8)(q) \ln(9)) \\
\approx (.95916) + (1 - q)(3.75308) + (q)(1.75778) \\
= (4.71224) - (1.99530)q
\]

He should choose to settle out of court if and only if this expression is less than his certain utility of \( \ln(45) \approx 3.80666 \) which can be realized by choosing to settle. That is, he should settle out of court if and only if:

\[
(4.71224) - (1.99530)q < 3.80666 \\
.90558 < (1.99530)q \\
q > .45386
\]

1E. Again supposing \( q = .75 \), Lawrence’s expected utility (as a function of \( p \)) from choosing “don’t settle” can be expressed as:

\[
(1 - p) \ln(121) + (p)(.25) \ln(109) + (.75) \ln(9) \\
\approx (4.79579)(1 - p) + (p)(1.17284 + 1.64792) \\
= (4.79579) - (1.97503)p
\]

He should choose to settle out of court if and only if this expression is less than his certain utility of \( \ln(45) \approx 3.80666 \) which can be realized by choosing to settle. That is, he should settle out of court if and only if:

\[
(4.79579) - (1.97503)p < 3.80666 \\
.98913 < (1.97503)p \\
p > .50082
\]

1F. More generally (in terms of both \( p \) and \( q \)), his expected utility from choosing “don’t settle” can be expressed as:

\[
(1 - p) \ln(121) + (p)(1 - q) \ln(109) + (p)(q) \ln(9) \\
= \ln(121) - p(\ln(121) - \ln(109)) - pq(\ln(109) - \ln(9)) \\
= \ln(121) - p \ln\left(\frac{121}{109}\right) - pq \ln\left(\frac{109}{9}\right)
\]

He should choose to settle out of court if and only if this expression is less than his certain utility of \( \ln(45) \) which can be realized by choosing to settle. That is, he should settle out of court if and only if:

\[
\ln(121) - p \ln\left(\frac{121}{109}\right) - pq \ln\left(\frac{109}{9}\right) < \ln(45) \\
\ln\left(\frac{121}{45}\right) < p \ln\left(\frac{121}{109}\right) + pq \ln\left(\frac{109}{9}\right)
\]

which is approximately

\[
.98913 < (.10444)p + (2.49412)pq
\]
2A. By Bayes’ Rule, your updated beliefs (after getting the report from “The Veblen Group”) would be: 
\[ P_r(h \mid H^*) = \frac{.225}{.225 + .0375} \approx .85714, \quad P_r(l \mid H^*) = \frac{.0375}{.225 + .0375} \approx .14286, \]
\[ P_r(l \mid F^*) = \frac{.7125}{.7125 + .025} \approx .96610, \quad \text{and} \quad P_r(h \mid F^*) = \frac{.025}{.7125 + .025} \approx .03390. \] 
Thus, if they were to report “Hit,” your expected payoff would be:
\[ \approx (.85714)(1,200,000) + (.14286)(300,000) - 400,000 = 671,426. \]
Since this is positive, you would want to bring the product to market following a report of “Hit” from “The Veblen Group.” Similarly, if they were to report “Flop,” then your expected payoff would be:
\[ \approx (.03390)(1,200,000) + (.96610)(300,000) - 400,000 = -69,490. \]
Since this is negative, you would not want to bring the product to market following a report of “Flop” from “The Veblen Group.”

2B. Recognize, “The Veblen Group” will report “Hit” with probability 
\[ P_r(H^*) = \frac{.225 + .0375}{.225 + .0375} = .2625 \]
and will report “Flop” with probability 
\[ P_r(F^*) = \frac{.7125 + .025}{.7125 + .025} = .7375. \] 
Thus, your expected payoff from hiring “The Veblen Group” would be:
\[ (2625)(671,426) + (7375)(0) - 75,000 = 101,249.325. \]

2C. By Bayes’ Rule, your updated beliefs (after getting the report from “Kwality Research”) would be: 
\[ P_k(h \mid H^*) = \frac{.2}{.2 + .3} = .4, \quad P_k(l \mid H^*) = \frac{.3}{.2 + .3} = .6, \quad P_k(l \mid F^*) = \frac{.45}{.05 + .45} = .9, \quad \text{and} \quad P_k(h \mid F^*) = \frac{.05}{.05 + .45} = .1. \] 
Thus, if they were to report “Hit,” your expected payoff would be:
\[ = (.4)(1,200,000) + (.6)(300,000) - 400,000 = 260,000. \]
Since this is positive, you would want to bring the product to market following a report of “Hit” from “Kwality Research.” Similarly, if they were to report “Flop,” then your expected payoff would be:
\[ = (.1)(1,200,000) + (.9)(300,000) - 400,000 = -10,000. \]
Since this is negative, you would not want to bring the product to market following a report of “Flop” from “Kwality Research.”

2D. Recognize, “Kwality Research” will report “Hit” with probability 
\[ P_r(H^*) = .2 + .3 = .5 \]
and will report “Flop” with probability 
\[ P_r(F^*) = .05 + .45 = .5. \] 
Thus, your expected payoff from hiring “Kwality Research” would be:
\[ (.5)(260,000) + (.5)(0) - 2,000 = 128,000. \]

2E. From the answers to (2B) and (2D), we see that hiring “Kwality Research” results in a greater expected payoff than hiring “The Veblen Group.” Further, since the expected payoff from introducing the product after hiring “Kwality Research” is positive, we know that this is a better choice than not bringing the product to market. Finally, recognize that the expected payoff from bringing the product to market without hiring either firm is:
\[ (.25)(1,200,000) + (.75)(300,000) - 400,000 = 125,000. \]
Thus, hiring “Kwality Research” results in the greatest expected profit of the four available options.
3A. Draw the decision tree that Teresa faces.

![Decision Tree Diagram]

3B. Teresa’s expected utility from choosing “Enter” is

\[(p)(120) + (1 - p)(60) = 60p + 60\]

whereas she can achieve a certain utility of 80 from choosing “Don’t Enter.” Thus, she will choose “Enter” if and only if:

\[60p + 60 > 80\]
\[60p > 20\]
\[p > \frac{2}{6} = \frac{1}{3} \approx 0.3333\]

3C. The firm owners desire to maximize expected profit. Expected profit from “Enter” is

\[(p)(1,440,000) + (1 - p)(360,000) = (1,080,000)p + 360,000\]

whereas a profit of $640,000 can be realized by choosing “Don’t Enter.” Thus, the firm owners would prefer “Enter” if and only if:

\[(1,080,000)p + 360,000 > 640,000\]
\[(1,080,000)p > 280,000\]
\[p > \frac{280,000}{1,080,000} = \frac{7}{27} \approx 0.25926\]

3D. Based upon the answers to (3B) and (3C), for any \(0.25926 < p < \frac{1}{3} \approx 0.3333\), Teresa will choose “Don’t Enter” even though the firm owners would prefer that she choose “Enter.”
4A. If Yawges pursues R&D sequentially, their two options are:

Thus, the expected payoff from pursuing “EE R&D” first is:

\[
(0.15)(-120,000 - 5x) + (0.0425)(-300,000 - 10x) + (0.8075)(700,000 - 10x)
\]

\[
= -18,000 - 0.75x - 12,750 - 0.425x + 565,250 - (8.075)x \\
= 534,500 - 9.25x
\]

which, for \( x = 7,500 \), becomes $465,125.

Whereas the expected payoff from pursuing “ME R&D” first is:

\[
(0.05)(-180,000 - 5x) + (0.1425)(-300,000 - 10x) + (0.8075)(700,000 - 10x)
\]

\[
= -9,000 - 0.25x - 42,750 - 1.425x + 565,250 - (8.075)x \\
= 513,500 - 9.75x
\]

which, for \( x = 7,500 \), becomes $440,375.

Thus, of these two options, the better choice is to pursue “EE R&D” first.
4B. If they pursue both types of R&D simultaneously, then:

which leads to an expected payoff of:

\[(.8075)(700,000 - 5x) + (.1925)(-300,000 - 5x)\]

which, for \(x = 7,500\), becomes $470,000. Thus, their best choice is to pursue “ME R&D” and “EE R&D” simultaneously.

4C. If instead \(x = 5,500\), then the expected payoff from pursuing “EE R&D” first would be: \(534,500 - 9.25x = 483,625\). Further, the expected payoff from pursuing “ME R&D” first would be: \(513,500 - 9.75x = 459,875\). So, of these two options, pursuing “EE R&D” first is still the better choice. However, the expected payoff from pursing both phases of R&D simultaneously is now \(507,500 - 5x = 480,000\), which is no longer the greatest expected payoff of the three options. Thus, for this lower value of \(x\), the best choice is to now pursue “EE R&D” first.