Endogenous Information Quality in Common Value Auctions*

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ABSTRACT

This paper examines a two stage game where information is endogenously acquired prior to single object common value auctions. In stage one, bidders choose the quality of their information service. Stage two then examines strategic behavior in bonus bid and royalty rate auctions, conditional on stage one information. Comparisons are drawn between the information quality selected at each auction, inherent inefficiencies, and the resulting revenues generated. It is shown that while the standard revenue advantage of the royalty rate auction over the bonus bid auction is reduced, the royalty rate auction creates fewer inefficiencies than does the bonus bid auction when costly information is endogenously chosen.
1 Introduction

The theory of auctions has received a great deal of attention in the economic literature of the past several decades. Beginning with Vickrey’s (1961) observations on revenue equivalence, theorists have developed a well founded framework in which to examine the behavior of auction participants. The usual focus has been on derivation of the Bayes-Nash equilibrium in various auctions and subsequent comparisons of those auctions in terms of efficiency and revenue generation. Despite a plenitude of results, a crucial aspect of the bidding process has been greatly ignored; information acquisition prior to the actual auction. This paper treats the information acquisition process as endogenous and examines the affects on established results.

The seminal work on single object auctions is Milgrom and Weber (1982). Their model of affiliated values encompasses the two primary paradigms on which the auction literature has been built; those of independent private (IPV) and common (CV) values. These paradigms are most commonly distinguished from one another based on the structure of the underlying uncertainty that bidders face. In the IPV paradigm, uncertainty arises solely from the privacy of individual bidder valuations. Alternatively, uncertainty in the common values framework takes on two forms. First, and similar to the IPV case, bidders receive privately known signals concerning the true

\footnote{Each bidder in the IPV setting has a potentially different valuation for the object that is privately known. Alternatively, the value of the object in a common values setting has the same ex post realization, independent of who wins the auction.}
value of the object. But, unlike the IPV case, bidders do not know the true value of the object being sold. The model considered in this paper explicitly examines bidders’ efforts to reduce the second form of uncertainty in the common values setting.

The specific environment we consider is the “mineral rights” model (see Milgrom and Weber (1982)) in which a single tract of land containing an unknown amount of mineral deposits is auctioned to a group of bidders. We opt for this model for several reasons. First, our results are easily interpreted in this context. Second, analytic solutions in each stage of the game are readily obtained in this framework. Third, our results are applicable to real world institutions such as OCS and US Forest Service auctions, where pre-auction information acquisition is a standard practice. Finally, the model provides a number of established results when information quality is exogenously determined, thus providing a benchmark to which our results can be compared.

We focus on bonus bid and royalty rate auction forms.\textsuperscript{2} The bidding variable in the bonus bid auction is the total payment which a participant will make to the seller for the right to extract the mineral deposits. The bidding variable in the royalty rate auction is a per unit extraction charge payed to the seller. Previous work by Milgrom and Weber (1982) and Riley (1982) tells us that the important difference between the two payment rules

\textsuperscript{2}While we explicitly focus on only two auctions, our results generalize to include second price and other auction forms under our distributional assumptions.
is that the price paid in the bonus bid auction is not dependent on the ex
post realization of the tract, while the price paid in the royalty rate auction
is. This difference leads to expected revenue dominance of the royalty rate
auction over the bonus bid auction for two reasons. First, Milgrom and
Weber (1982) establish that linking the price paid to outside information is
revenue enhancing. Second, Riley (1982) shows that revealing information
subsequent to the auction also has a revenue enhancing effect.

Another common grounds for comparing auctions is efficiency. However,
within the common values framework, these comparisons are usually mun-
dane. The reason is simple, in a common values environment, the ex post
value of the tract is independent of the winner’s identity, guaranteeing an
efficient allocation. However, allowing for (costly) information acquisition
opens the door for potential inefficiencies. Since the allocative efficiency of
the auction is independent of the winner, individual efforts by bidders to be-
come better informed are purely rent seeking activities (see Matthews (1984),
Reece (1978), Tiesberg (1977)). Therefore, those auctions that induce high
levels of costly information acquisition will be less efficient than those that
induce low levels.

While the revenue results of Milgrom and Weber and Riley are illuminat-
ing, their revenue rankings are based on the same level of information quality
being taken into the alternative auctions. Yet, there is no theoretical justifi-
cation for such an assumption. The goal of our paper is to derive the level of
information acquisition in bonus bid and royalty rate auctions and compare
the resulting efficiency and revenue generating properties of each. Our task is made easier by the previous work of Matthews’ (1984) concerning bonus bid auctions. In order to apply his results, we adopt the same two stage format that he considered. In stage one, bidders choose the level of information quality to be taken into the auction and then bid in the auction in stage two. Section 2 reviews the details of the model, including the informational assumptions that we maintain throughout.

Bid strategies with exogenous information quality are important to us for they are the first step in isolating the sub-game perfect equilibrium. In addition, they provide a benchmark against which our endogenous information results can be compared. As such, the symmetric Nash equilibrium bidding strategy of each auction mechanism is derived in Section 3. Using these equilibrium strategies, expected seller revenues from each of the auctions are computed. It is shown that when information is treated exogenously, the expected revenue generated by the royalty rate auction dominates that generated by the bonus bid auction. As we consider a broader class of profit functions, this is an extension of Riley’s (1982) revenue ranking.

In Section 4, a participant’s choice of information quality is examined. The methodology is to examine stage one, given the equilibrium bidding behavior that will follow in stage two. At this point we break from generality, specifying the functional form of the underlying distribution of the object’s true value, as well as, the conditional distribution of bidder signals. The assumption on the underlying distribution of the object’s true value is made
solely for the sake of tractability. However, by assuming that the distribution of signals is conditionally uniform, we eliminate the revenue enhancing effects of the linkage principle, thereby isolating the effects of tying the price paid to ex post information. We find that the equilibrium message service of the bonus bid auction is always more informative than the corresponding equilibrium message service of the royalty rate auction. The intuition for this result is straightforward. Participants in a royalty rate auction share the risk associated with the underlying uncertainty with the seller. Therefore, informativeness is worth less in the royalty rate auction than in the bonus bid auction.

Section 4 develops and utilizes two basic results. First, when the same level of information is taken into each auction, the royalty rate auction dominates the bonus bid auction in terms of revenue generation. Second, when information quality is a choice variable, more information is acquired in the bonus bid auction. Next, we establish that higher quality information leads to higher revenues than does lower quality information. In revenue generating terms, this means that the effects of information acquisition work in favor of the auction that does not link the price paid to ex post information. The result is that although the royalty rate auction still out performs the bonus bid auction, the difference in the revenues generated by the two auctions is lessened. However, the increased revenue from the bonus bid auction comes at the expense of over-investment in information and a loss of efficiency relative to the royalty rate auction. Finally, conclusions and possible extensions
are offered in Section 5.

2 The Environment

A single tract of land containing an unknown quantity of mineral deposits is auctioned to \( n \) risk neutral bidders. Each bidder receives a conditionally independent signal regarding the “true” quantity \((q)\) of deposits on the tract. This true value is determined by nature prior to the game, which unfolds as follows.

**Stage 1** Players simultaneously choose the quality of their message service \( \theta_i \) and pay according to the linear cost function \( TC(\theta_i) = C\theta_i \). Players then privately realize their signals \( s_i \) from the conditional distribution function \( F_{\theta_i}(s|q) \).

**Stage 2** Observing the quality decision of their opponents \( \theta_j \forall j \neq i \), bidders compete for the mineral rights at auction.

When analyzing the stage one choice of information quality, we will lean heavily on the concept of *more informative* from the economics and statistics literature on information systems.

**Definition 1** An information service \((A)\) is said to be more informative than another information service \((B)\) for a given set of prior beliefs \( \pi \) if \( A \) provides an agent with higher expected utility than \( B \) regardless of the agent’s utility function (see Hirschliefer and Riley (1982)).
Marshak and Miyasawa (1968) tell us that more informative provides only a partial ordering of information systems. It is certainly possible to have two information systems that can not be ranked according to more informative. Blackwell (1951, 1953) has derived conditions under which information systems in discrete probability space can be ranked according to more informative. This is the well known garbling condition. In addition, Matthews (1984) demonstrates similar conditions for continuous probability spaces.

In the absence of any game theoretic considerations, an agent will always prefer to base decisions upon a more informative signal. However, several authors have considered the role of improved information in a game theoretic environment. Green and Stokey (1980) and Crawford and Sobel (1982) consider more informative information systems in the context of a principle agent problem. In each, more informative information systems do not necessarily improve the welfare of the players since all players engage in the information collecting process.

2.1 Informational Assumptions

**Assumption 1** It is common knowledge that the random variable $q$ is distributed continuously over $(q, \bar{q})$ according to the continuously differentiable distribution function $G(q)$ which has a corresponding density function $g(q)$.

**Assumption 2** The post-auction profit of the winning bidder, as a function of $q$, is given by a non-decreasing function, $\pi(q)$, that is known to be common
among all the bidders.

Assumptions 1 and 2 imply that the random variable $q$ is the sole source of underlying uncertainty. In addition, the following interpretation is given for Assumption 2. The function can be written as $\pi(q) = (P - AC(q))q$ where $AC(q)$ is the common average cost of extracting $q$, and $P$ is the known selling price for a unit of the extracted resource. Assumption 2 is equivalent to assuming that $AC(q)$ is non-increasing which has the interpretation that high fixed costs are associated with extraction of the resource, a reasonable assumption in light of the mineral rights model.

Participants may also subscribe to a message service that provides a conditionally independent signal $s$ that is correlated with the true realization $q$. As Matthews indicates, some message services are ex ante more informative than others. For example, one service may provide a strictly finer information partition than another. In modelling the informativeness of a message service, we opt for that introduced by Matthews, defining $\theta$ as a parameter of the distribution from which signals are drawn that directly measures the quality of the message service. Specifically, a signal from a message service of quality $\hat{\theta}$ is said to be more informative than a signal from a message service of quality $\theta$ if and only if $\hat{\theta} > \theta$.

**Assumption 3** Given any realization $q$ and a chosen level of information quality $\theta$, signals are drawn from the continuously differentiable distribution function $F_{\theta}(s|q) = F(s|q)^{\theta}$ that is defined over the range $(q, q)$ and has cor-
responding density function \( f_\theta(s|q) \) where \( F(s|q)^\theta \) orders more informative.\(^3\)

The implication of Assumption 3 is that signals are biased in the sense that every signal must lie below the true realization \( q \). In this context, a signal may be interpreted as a sample of resources taken from the tract. This sample can, in turn, be used by the bidder formulate an estimate concerning the value of \( q \). The restrictions on the bounds of the distributions then become obvious. It would be impossible to draw a sample that exceeded the actual size of the deposits present. Thus, signals must lie below the true quantity. Yet, it is certainly possible to infer a quantity estimate (via Bayes Law) that exceeds the true quantity.

3 Exogenous Information Acquisition

The first step in solving for the sub-game perfect equilibrium is solving for the stage two bid strategies, given the stage one informational decisions, whatever they may be. This boils down to solving the case of exogenous information and then implementing the backward induction algorithm. In this section, we examine the bidder’s bidding decision in each auction given a fixed level of information quality. We remind the reader, however, that the fixed quantity in each auction will be determined in stage one and is thus not necessarily fixed across auctions.

\(^3\)In Section 4 we assume the function form of \( F_\theta(s|q) \) to be \( F(s|q)^\theta = (s/q)^\theta \) over the range \((0, q)\) which Matthews (1984) has proved orders more informative.
The model examined here is a variant of that proposed by Matthews (1984). For sake of comparison to other auction formats, his results, and many others, are developed here within our framework. The ex ante symmetry of the game allows us to consider the problem faced by a single bidder without loss of generality. The first step is the construction of the bidder’s estimate of the unknown quantity $q$ given their signal $s$ and the quality of the information service $\theta$. Upon receiving their signal, a bidder updates their prior $g(q)$ via Bayes Law, giving

$$g(q|s, \theta) = \frac{\theta f(s|q)F(s|q)\theta^{-1}g(q)}{\int q \theta f(s|w)F(s|w)\theta^{-1}g(w)dw}. \quad (3.1)$$

It follows that the expected value of the tract to a player with signal $s$ drawn from a message service of quality $\theta$ is,

$$v(s, \theta) = \int g(q|s, \theta) dq. \quad (3.2)$$

However, any bid function that uses the expectation given by equation (3.2) is sub-optimal. A sophisticated bidder will only be concerned with value of the object conditional upon winning the auction. That is, the expectation given by equation (3.2) does not take into account the information that winning conveys. In other words, any bid based upon the above expectation would be susceptible to the winners curse.

A more sophisticated estimate of the value of the object conditioned on winning the auction is given by,
\[ v(s, \phi_n) = \int_s^\pi(q) g(q|s, \phi_n) dq, \]  

(3.3)

where \( \phi_n = n\theta \) is a measure of the aggregate information acquired by all bidders. That is, \( v(s, \phi_n) \) accounts for the fact that the winning bidder will have the highest of \( n \) draws from \( F_\theta(s|q) \). Therefore, the winning bidder’s signal will follow a distribution given by \( F_{\phi_n}(s|q) \). In this way, equation (3.3) adjusts for the winners curse. Of course, equation (3.3) is not imposed upon the bidders. Rather, it will emerge as a component of the equilibrium bid function.

### 3.1 The Bonus Bid Auction

The bidder’s problem in the bonus bid auction is simply to choose a bid that maximizes expected surplus. Consider bidder \( i \)'s choice of a bid \( B \) that maximizes expected surplus given that all of the other bidders are using some monotonic bid strategy \( B^*(s) \) with inverse \( \psi(B) \). The goal is to find a specific functional form of \( B^*(s) \) such that when all other bidders use that strategy, bidder \( i \)'s best response is to use it as well. To wit, this function is defined as,

\[
B^*(s) = \arg \max_b \int_s^\pi F(\psi(B)|q)^{\phi_{-i}} (\pi(q) - B) g(q|s, \theta) dq, \tag{3.4}
\]

where \( \phi_{-i} = (n - 1)\theta \) is a measure of the aggregate information acquired by all players other than \( i \).
The interpretation of equation (3.4) is straightforward. The first component of the integrand represents the probability that all opponents’ signals are lower than i’s signal. This is exactly the probability of winning the object if the other players adopt the equilibrium strategy. The second component is simply the profit from winning the object with a bid of \( B \). Finally, these terms must be integrated over the probability of each possible realization of \( q \).

**Proposition 1** The unique monotonic symmetric equilibrium bidding strategy derived from equation (3.4) is,

\[
B^*(s) = \int_q^s v(t, \phi_n) h(t|s) dt, \tag{3.5}
\]

where \( h(t|s) \) is the conditional density of the second highest signal \( t \) given the highest signal.\(^4\)

**Proof.** The first order condition obtained from differentiating equation (3.4) with respect to \( B \) is

\[
\int_s^q \psi'(B) \phi_{-i} F(\psi(B)|q) \phi_{-i}^{-1} f(\psi(B)|q) (\pi(q) - B) g(q|s, \theta) dq = 0. \tag{3.6}
\]

\(^4\)Under the assumption of conditionally uniform signals \( h(t|s) \) is given by \( h(t|s) = \phi_i \left( \frac{f(t|q)}{f(s|q)} \right)^{\phi_{-i}^{-1}} \left( \frac{f(t|q)}{f(s|q)} \right) \).
Substituting for \( g(q|s, \theta) \) using equation (3.1) and evaluating at the symmetric equilibrium gives,

\[
\int_s^\overline{s} \left[ \frac{1}{B^*(s)} \phi_i F(s|q)^{\phi-i} f(s|q) (\pi(q) - B(s)) - F(s|q)^{\phi-i} \right] F(s|q)^{\phi_i-1} f(s|q) g(q) dq = 0.
\] (3.7)

To complete the proof, we show that if \( B^*(s) \), obtained from differentiating equation (3.5) with respect to \( s \), is substituted in equation (3.7), then the first order condition is satisfied.

Differentiating equation (3.5) with respect to \( s \), it is easily shown that

\[
B^*(s) = \phi_i F(s|q)^{-1} f(s|q) (v(s, \phi_n) - B^*(s))
\] (3.8)

Substituting equation (3.8) into equation (3.7) gives

\[
\int_s^\overline{s} \left[ \frac{\pi(q) - B^*(s)}{v(s, \phi_n) - B^*(s)} - 1 \right] F(s|q)^{\phi_n-1} f(s|q) g(q) dq = 0,
\]

which implies,

\[
\int_s^\overline{s} \left[ \frac{\pi(q) - v(s, \phi_n)}{v(s, \phi_n) - B(s)} \right] F(s|q)^{\phi_n-1} f(s|q) g(q) dq = 0.
\]

Rearranging, dividing by \( \int_s^\overline{s} F(s|w)^{\phi_n-1} f(s|w) g(w) dw \), and multiplying by \( v(s, \phi_n) - B^*(s) \) gives

\[
\frac{\int_s^\overline{s} \pi(q) F(s|q)^{\phi_n-1} f(s|q) g(q) dq}{\int_s^\overline{s} F(s|w)^{\phi_n-1} f(s|w) g(w) dw} - v(s, \phi_n) = 0
\] (3.9)
Noticing that the first term in equation (3.9) equals \( v(s, \phi_n) \) by equation (3.3) completes the proof. \textit{QED}

The intuition behind the equilibrium bid function is well known. The bid of the player with the highest signal exactly equals the expectation of the surplus of the player with the second highest signal, conditional on it being the second highest.

3.2 The Royalty Rate Auction

The bidding variable in the royalty rate auction is the price to be paid per unit of the resource extracted.\(^5\) The bidder’s objective is to maximize expected surplus through a choice of a bid \( R \) while all other participants use the strategy \( R^*(s) \) with inverse \( \eta(R) \). The goal is to find a functional form of \( R^*(s) \) such that it is also a best response from bidder \( i \). The equilibrium strategy is then defined by,

\[
R^*(s) = \arg \max_\pi \int_q \eta(R) |q|^{\phi_i} (\pi(q) - Rq) g(q|s, \theta) dq. \quad (3.10)
\]

The objective function is easily interpreted. The first term is the probability of winning given a bid of \( R \). The second term is simply profit from winning the object with a bid of \( R \). Finally, these terms must be integrated over the probability of each possible realization of \( q \).

\(^5\)This mechanism is analogous to the scale sales used by the US Forest Service where participants bid a price per thousand board foot for each species rather than a single price for the entire timber tract. See Baldwin et al. (1997) for a complete description of the bidding procedures used by the US Forest Service. In addition, the Department of the Interior has held exactly one Outer Continental Shelf lease sale via the royalty rate mechanism. For details concerning OCS auctions, see Engelbrecht-Wiggans (1980).
Proposition 2  The unique monotonic symmetric equilibrium bid function of the royalty rate auction is given by,

\[
R^*(s) = \int_q^s \left( \frac{v(t, \phi_n)}{\tilde{q}(t, \phi_n)} \right) h(t|s) dt, \tag{3.11}
\]

where \(\tilde{q}(t, \phi_n) = \int_t^\infty qg(q|t, \phi_n) dq\) is the expectation of \(q\) given the second highest signal \(t\) and aggregate information \(\phi_n\).

Proof. To begin, let \(k(s, \phi_n) = \frac{v(t, \phi_n)}{\tilde{q}(t, \phi_n)}\). The first order condition obtained from differentiating equation (3.10) with respect to \(R\) is

\[
\int_s^\infty \eta'(R) \phi^{-i} F(\eta(R)|q)^{\phi^{-i}-1} f(\eta(R)|q) (\pi(q) - Rq) g(q|s, \theta) dq \tag{3.12}
\]

\[\quad - \int_s^\infty qF(\eta(R)|q)^{\phi^{-i}} g(q|s, \theta) dq = 0.
\]

Substituting for \(g(q|s, \theta)\) using equation (3.1) and evaluating equation (3.12) at the symmetric equilibrium gives

\[
\int_s^\infty \left[ \frac{1}{R^*(s)} \phi^{-i} F(s|q)^{\phi^{-i}-1} f(s|q) (\pi(q) - R(s)q) - qF(s|q)^{\phi^{-i}} \right] F(s|q)^{\phi^{-i}} f(s|q) q(q) dq = 0. \tag{3.13}
\]

To complete the proof, we show that if \(R^*(s)\), obtained from differentiating equation (3.11) with respect to \(s\), is substituted into equation (3.13), then the first order condition is satisfied.

Differentiating equation (3.11) with respect to \(s\), it is easily shown that

\[
R^*(s) = \phi^{-i} F(s|q)^{-1} f(s|q) (k(s, \phi_n) - R^*(s)) \tag{3.14}
\]
Substituting equation (3.14) into equation (3.13) gives

\[
\int_s \pi(q) - R^*(s)q \left( \frac{k(s, \phi_n)}{k(s, \phi_n) - R^*(s)} - q \right) F(s|q)^{\phi_n - 1} f(s|q)g(q) dq = 0,
\]

which implies,

\[
\int_s \pi(q) - k(s, \phi_n)q \left( \frac{k(s, \phi_n)}{k(s, \phi_n) - R^*(s)} - q \right) F(s|q)^{\phi_n - 1} f(s|q)g(q) dq = 0.
\]

Multiplying by \( k(s, \phi_n) - R(s) \) and rearranging gives

\[
\int_s \pi(q) F(s|q)^{\phi_n - 1} f(s|q)g(q) dq = k(s, \phi_n) \int_s q F(s|q)^{\phi_n - 1} f(s|q)g(q) dq,
\]

or,

\[
k(s, \phi_n) = \frac{\int_s \pi(q) F(s|q)^{\phi_n - 1} f(s|q)g(q) dq}{\int_s q F(s|q)^{\phi_n - 1} f(s|q)g(q) dq}.
\]

By the definition of \( k(s, \phi_n) \), the left hand side of the LHS of the last equation equals the RHS. \( QED \)

Recall that the symmetric equilibrium bid in the bonus bid auction is the expectation of the second highest valuation for the object conditional upon having the highest valuation. \( R^*(s) \) has a similar interpretation upon realizing that \( \tilde{v}(t, \phi_n) \) is the relevant valuation for buyers at a royalty rate auction. To see this recall that the profit function \( \pi(q) \) can be written as \( (P - AC(q))q \). In this case, the surplus to the winning bidder at a royalty rate auction is \( (P - AC(q))q - R^*(s)q \). Because \( q \) factors out of the royalty rate surplus, bidders will be concerned with average profit and not total profit.
as in the bonus bid auction. Hence, the expectation of average profit is the relevant statistic in the royalty rate auction.

### 3.3 Additional Assumptions

In order to calculate the auctioneer’s expected revenue in the two auctions, we are forced to make additional assumptions for the sake of tractability. These assumptions will also aid in the implementation of the backward induction algorithm used to analyze the choice of information quality.

**Assumption 4** *The functional form of the conditional distribution is* \( F_\theta(s|q) = F(s|q)^\theta = (s/q)^\theta \) *over the range* \((0, q)\).*

**Assumption 5** *The functional form of the prior density with regard to the random variable* \( q \) *is restricted to the Pareto family* \( g(q) = \alpha q^{-(\alpha+1)} \) *for* \( q \) *in the range* \((1, \infty)\), \( \alpha \geq 1 \).

**Assumption 6** *The form of the profit function for the winning firm is restricted to the family given by* \( \pi(q) = q^\gamma \) *for* \( \alpha > \gamma > 0 \).

Under Assumption 4, \( \theta \) does indeed order *more informative* (see Matthews (1984) for proof). The intuition is helpful and straightforward. Suppose that \( \theta \) represents the number of draws taken from the conditional distribution \( F(s|q) \). Then \( F(s|q) \) is simply the conditional uniform distribution while \( F_\theta(s|q) \) is the distribution of the highest order statistic from a sample of size \( \theta \). Because the highest order statistic of a sample from the uniform
distribution is a sufficient statistic for the unknown upper bound \( q \), the most meaningful signal from the \( \theta \) draws is the highest. Clearly, the highest order statistic from a larger sample is more informative than that from a smaller sample.

Assumption 5 is useful in the present framework since the Pareto density corresponds to a uniformly distributed random variable with unknown upper support. This seems plausible for the mineral rights model we are considering and yields tractable solutions. Finally, under Assumption 6, the ex ante expected value of the tract is \( \frac{\alpha}{(\alpha - \gamma)} \) which explains the restriction on \( \gamma \).

Under Assumption 4, 5, and 6, closed form expressions for the equilibrium bid functions are,

\[
B^*(s) = \frac{(\phi_n + \alpha)}{(\phi + \gamma)} \left( \gamma m(s)^{-\phi_{-i}} + \phi_{-i} m(s)^{\gamma} \right) \tag{3.15}
\]

and

\[
R^*(s) = \frac{(\phi_n + \alpha - 1)}{(\phi_{-i} + \gamma - 1)(\phi_n + \alpha - \gamma)} \left( (\gamma - 1) m(s)^{-\phi_{-i}} + \phi_{-i} m(s)^{\gamma - 1} \right) ,
\tag{3.16}
\]

where \( m(s) = \max(1, s) \) accounts for the behavior of bidders who receive signals in the interval (0,1). This adjustment is necessary since bidders that receive signals in the interval (0,1) realize that the lower support of the mineral rights distribution is one and thus update their signal to the lowest possible realization of \( q \).
3.4 Expected Seller Revenue

This section considers the expected seller revenue generated by each mechanism, given that the equilibrium bid functions derived above are followed. As described in the introduction, Riley (1982) has shown that the expected seller revenue from a royalty rate auction dominates the corresponding revenue from the bonus bid auction if information quality is exogenously determined. This section extends Riley’s result to the case of strictly convex and strictly concave profit functions since his result was obtained implicitly for linear profit functions.

Define $\text{Rev}_B(\phi_n)$ as the expected seller revenue at the bonus bid auction and $\text{Rev}_R(\phi_n)$ as the expected seller revenue at the royalty rate auction. In general,

$$\text{Rev}_B(\phi_n) = \int_{q}^{\hat{q}} \int_{\hat{s}}^{q} B^*(s) \phi_n F(s|q) \phi_n^{-1} f(s|q) g(q) dsdq \quad (3.17)$$

$$\text{Rev}_R(\phi_n) = \int_{q}^{\hat{q}} \int_{\hat{s}}^{q} R^*(s) q \phi_n F(s|q) \phi_n^{-1} f(s|q) g(q) dsdq. \quad (3.18)$$

**Proposition 3** If information quality is exogenously given as $\phi_n$, then $\text{Rev}_R(\phi_n) \geq \text{Rev}_B(\phi_n)$.

**Proof:** Under Assumptions 4, 5, and 6, closed form expressions for equations (3.17) and (3.18) can be computed as,

$$\text{Rev}_B = \frac{\alpha}{(\alpha - \gamma)} \left[ \zeta + \frac{\phi_n}{(\phi_{-i} + \gamma)(\phi_n + \alpha - \gamma)} \left( \frac{\gamma (\alpha - \gamma)}{(\phi_{-i} + \alpha)} + \phi_{-i} \right) \right] \quad (3.19)$$
\[ \text{Rev}_R = \frac{\alpha}{(\alpha - \gamma)} \left[ \zeta + \frac{\phi_n}{(\phi_{-i} + \gamma - 1)(\phi_{n} + \alpha - \gamma)} \left( \frac{(\gamma - 1)(\alpha - \gamma)}{(\phi_{-i} + \alpha - 1)} + \phi_{3.20} \right) \right] \]

where \( \zeta = \frac{(\alpha - \gamma)}{\phi_{n} + \alpha - \gamma} \).

Algebraic manipulation shows the revenue differential is,

\[ \text{Rev}_R - \text{Rev}_B = \frac{\alpha \phi_n}{(\phi_{-i} + \alpha - 1)(\phi_{-i} + \alpha)(\alpha - \gamma)}. \quad (3.21) \]

Clearly the differential is non-negative. \( QED \)

Recall that the ex ante expected value of the tract is given by \( \frac{\alpha}{\alpha - \gamma} \). It can easily be shown that the entire bracketed term in each of the revenue equations (3.19) and (3.20) ranges between zero and one. Therefore, maximal seller revenue is achieved when all of the ex ante expected surplus is extracted from the buyers. This full extraction occurs as \( \theta \) approaches both zero and infinity. For all other values of \( \theta \), less than full extraction occurs.

The intuition for this result is straightforward. As \( \theta \) approaches zero, the participants have signals that are completely uninformative. That is, the posterior density is equal to the prior density. In this case, each of the risk neutral contestants is willing to pay any amount less than or equal to the ex ante expected value of the tract. In the spirit of Bertrand Competition, the contestants bid away all of the surplus to the seller. Alternatively, as \( \theta \) approaches infinity, the probability that any signal is different than the true underlying quantity approaches zero. In this case, all bidders know the true value of the tract with certainty. As each bidder is willing to bid up to the
true value, the ex ante expected revenue is the expected value of the tract.

4 Endogenous Information Quality

This section considers the stage one information decision, given the stage two equilibrium bid functions. A strategy for the game described in Section 2 is a pair \((\theta, B(s))\) where \(\theta\) is the stage one decision variable and \(b\) is the stage two decision variable. A sub-game perfect, symmetric equilibrium is therefore an \(n\)-tuple of pairs \((\theta^*, B^*(s))\), each element of which is a best response to the other \((n - 1)\) and \(B^*\) is as derived in Section 3. Imposing sub-game perfection is tantamount to fixing \(B^*(s)\) at some information quality level and examining stage one.

4.1 Information Quality and Bonus Bidding

Consider bidder \(i\)'s choice of \(\theta\), given that all other bidders adopt the strategy \(\theta^*\). Once again, we look for a functional form of \(\theta^*\) such that bidder \(i\)'s best response is to use that function as well. The ex ante expected surplus of a bidder with information of quality \(\theta\) is given by,

\[
U_B = \int_q F(s|q)^{(n-1)\theta^*} \left( \pi(q) - B^*(s) \right) \theta F(s|q)^{\theta - 1} f(s|q) g(q) ds dq - C\theta. 
\]

(4.22)

Under assumption 6, equation (4.22) can be expressed in the closed form,

\[
U_B = \frac{\alpha \gamma \theta}{(\alpha - \gamma)(\phi_n + \alpha)(\phi_m + \alpha - \gamma)} - C\theta 
\]

(4.23)
Notice that buyer surplus is increasing in the expected value of the tract as was seller revenue. Equation (4.23) can also be interpreted simply as the benefits of information acquisition minus the costs. Under this interpretation, it is clear that the marginal benefit of increased information is always positive. Furthermore, expected utility is globally concave in $\theta$ which ensures the sufficiency condition for maximization is satisfied.

Differentiating (4.23) yields the first order necessary condition,

$$C = \frac{\alpha \gamma (\phi_{-i} + \alpha - \gamma)}{(\alpha - \gamma)(\phi_{-i} + \alpha)(\phi_{-i} + \theta + \alpha - \gamma)^2}. \quad (4.24)$$

Evaluating (4.24) the first order condition at the symmetric equilibrium $\theta^*$ yields,

$$C = \frac{\alpha \gamma ((n - 1)\theta^* + \alpha - \gamma)}{(\alpha - \gamma)((n - 1)\theta^* + \alpha)(n\theta^* + \alpha - \gamma)^2}. \quad (4.25)$$

As the original problem is globally concave in $\theta$, equation (4.25) completely characterizes the symmetric equilibrium in information acquisition. Equation (4.25) defines a cubic polynomial which implies the existence of at most three real solutions. For most specifications of the exogenous parameters the equation has only one real solution. Unfortunately, the real solution is unmanageable from the standpoint of analytic tractability. Despite the absence of an explicit solution, a variety of interesting results are obtained from careful examination of the implicit solution defined by equation (4.25).
Proposition 4

\[ (i) \lim_{C \to 0} (\theta_B^*) = \infty \]

\[ (ii) \lim_{C \to \infty} (\theta_B^*) = 0 \]

**Proof:** As the left side of equation (4.25) approaches zero, the right side must also approach zero to maintain equality. The denominator of the right is a third degree polynomial in \( \theta^* \) while the numerator is only a first degree polynomial in \( \theta^* \). Both the numerator and the denominator can be divided by \( \theta^* \) to obtain,

\[
\frac{\alpha \gamma (n - 1) + \frac{(\alpha - \gamma)}{\theta^*}}{\left(\alpha - \gamma\right) \left( (n - 1) + \frac{(\alpha - 1)}{\theta^*} \right) (n\theta^* + \alpha - \gamma)^2}.
\]

(4.26)

As \( \alpha \geq 1, \alpha > \gamma > 0, \) and \( n > 1 \) are finite constants, equation (4.26) can only approach zero if \( \theta^* \) approaches infinity. This establishes part (i) of the proposition. The proof of part (ii) is equally straightforward and is omitted. *QED*

The intuition behind Proposition 4 is clear. As the cost of information acquisition becomes prohibitive, the equilibrium information quality degenerates. On the other hand, as the costs approach zero, the information quality approaches perfection.
4.2 Information Quality and Royalty Rate Bidding

This section derives the necessary and sufficient conditions for the symmetric equilibrium in information acquisition at the royalty rate auction. Again an \( n \)-tuple of strategy pairs \((\theta^*, R^*(s))\) that are mutual best responses to one another is sought. As in the bonus bid auction, the equilibrium bid function \( R^*(s) \) is valid for any symmetric realization of information \( \phi_n = n\theta \). Thus it will continue to be the equilibrium bid function for the special case of \( \phi^*_n = n\theta^* \). Therefore, only a condition for the determination of \( \theta^* \) is required.

The ex ante expected surplus of a participant at the royalty rate auction who chooses information quality \( \theta \), while all other bidders use the equilibrium strategy \( \theta^* \) is,

\[
U_R = \int_q^0 \int_s^q F(s|q)^{(n-1)}\theta^* (\pi(q) - R^*(s)q) \theta F(s|q)^{(\theta-1)} f(s|q)g(q)dsdq - C\theta.
\]

(4.27)

The closed form expression for equation (4.27), given the assumptions of the model is,

\[
U_R = \frac{\alpha(\gamma - 1)\theta}{(\alpha - \gamma)(\phi_{-i} + \alpha - 1)(\phi_n + \alpha - \gamma)} - C\theta.
\]

(4.28)

Notice that bidder surplus is increasing in the expected value of the tract and the marginal benefit of information acquisition is always positive. Since expected utility is globally concave in \( \theta \), we are assured that the sufficiency condition for a maximum will be satisfied.
The first order necessary condition obtained from equation (4.28) is,

\[ C = \frac{\alpha(\gamma - 1)(\phi_i + \alpha - \gamma)}{(\alpha - \gamma)(\phi_i + \alpha - 1)(\phi_n + \alpha - \gamma)^2}. \]  

(4.29)

Evaluating equation (4.29) at the symmetric equilibrium \( \theta^* \) yields,

\[ C = \frac{\alpha(\gamma - 1)(n - 1)\theta^* + \alpha - \gamma}{(\alpha - \gamma)(n - 1)\theta^* + \alpha - 1}(n\theta^* + \alpha - \gamma)^2. \]  

(4.30)

As the problem described by equation (4.28) is concave in \( \theta \), equation (4.30) completely characterizes the symmetric Nash equilibrium in information quality. Below, the counterpart of Proposition 4 for the royalty rate auction is presented.

**Proposition 5**

\((i)\lim_{C \to 0} (\theta^*_R) = \infty\)

\((ii)\lim_{C \to \infty} (\theta^*_R) = 0\)

The proof of Proposition 5 follows the proof of Proposition 4 and is omitted.

### 4.3 Equilibrium Information Quality

This section presents the main proposition of the paper. The information quality selected at the symmetric equilibrium of the bonus bid auction is compared with that selected at the corresponding equilibrium from the royalty rate auction. Although equations (4.25) and (4.30) are similar, the
conditions are not identical. Thus, the equilibrium information quality will differ across the auctions. The following Lemma is required to prove the Proposition 6.

**Lemma 1** Let \( Z_1 = f(x) \) and \( Z_2 = g(x) \) be two continuously differentiable, strictly decreasing functions satisfying \( f(x) < g(x) \) for all \( x \) over some domain \( (X^0, X) \). Then, the inverse mappings defined by \( f^{-1}(f(x)) = x \) and \( g^{-1}(g(x)) = x \) must satisfy: \( f^{-1}(z) < g^{-1}(z) \) for all \( z \) in the range of \( f(x) \).

The Lemma is easily verified by examining a graph in which two arbitrary functions satisfy the conditions of the remark. A formal proof is omitted.

**Proposition 6** Suppose that \( \alpha \geq 1, \ n > 1, \) and \( \alpha > \gamma > 0 \) are exogenous finite constants. Then,

\[
\theta^*_B(C) > \theta^*_R(C) \quad \forall \ C \text{ such that } 0 < C < \infty.
\]  (4.31)

**Proof:** In order to prove the proposition, the implicit solutions for the symmetric equilibria in information quality are rewritten as,

\[
C_B(\theta^*) = f(\theta^*)
\]
\[
C_R(\theta^*) = g(\theta^*).
\]

Under this interpretation, it is easy to show that \( C_B(\theta^*) > C_R(\theta^*) \) for any given \( \theta^* \). As both functions are strictly decreasing and continuously differentiable in \( \theta^* \), the conditions of Lemma 1 are satisfied. Therefore, the inverse
mappings $\theta_B(C) = f^{-1}(C)$ and $\theta_R(C) = g^{-1}(C)$ must satisfy equation (4.24) by Lemma 1.  \textit{QED}

The importance of Proposition 6 cannot be overstated. The underlying incentives for information acquisition are shown to be lower at the royalty rate auction than at the bonus bid auction for any choice of the parameters of the model. The intuition for this result is straightforward. Bidders in a royalty rate auction share the risk associated with the underlying uncertainty with the seller. Therefore, the bidders have less incentive to become informed about the true realization than a bonus bidding participant who does not share the risk with the seller.

In the context of this model, resources devoted toward information acquisition serve no productive purpose. This implies that one auction mechanism is more efficient than another if it induces less spending on information acquisition. Proposition 6, therefore reveals that the royalty rate auction is more efficient than the bonus bid auction.

5 Conclusion

This paper has presented an analytically tractable common value auction model that allows for an endogenous choice of information quality. Bidders’ strategic choices of information quality in conjunction with their optimal bidding strategies are derived for bonus bid and royalty rate auctions. When information is exogenously specified, the royalty rate auction dominates the bonus bid auction in terms of revenue generation since it links the price paid
with ex post information. Further, since the realized value of the object being auctioned is independent of the winner of the auction, allocative efficiency is a forgone conclusion.

Once the choice of information quality is modeled, we find that the equilibrium level of quality differs between the two auction forms. The fact that the royalty rate auction links the price paid by the winner to ex post information lessens bidder exposure to the winners curse versus a bonus bid auction. A direct result is that bidders have less incentive to acquire information in the royalty rate auction versus the bonus bid auction. In the context of the mineral rights auction, information acquisition is purely rent seeking behavior. As such, the royalty rate auction leads to a more efficient outcome since it devotes fewer resources to information acquisition.

The crux of the paper lies in the link between increased information quality, aggressiveness of bidding behavior, and increased auction revenues. In order to explain this, we begin by showing that lower information quality leads to lower bids. That is, while the link to ex post information makes bids more aggressive in the royalty rate auction, the information acquisition effect has the exact opposite effect. Thus, endogenous information acquisition produces a force that works against the revenue dominance of the royalty rate over bonus bid auctions found by Milgrom and Weber (1982) and Riley (1982). Unfortunately, the complexity of the first order conditions in stage one of the game does not allow for a definitive ranking of the two auctions. However, it has been established that the increased revenues in the bonus
bid auction comes at the expense of efficiency as more information is collected than prior to a royalty rate auction. Hence, the royalty rate auction is more efficient when information is endogenously chosen. This can be seen as support for the government’s use of royalty rate type auctions in timber sales by the US Forest Service and provides further reason for the royalty rate auctions to be used to sell commodities such as offshore oil leases.

References


