Incorporating Japanese Curriculum Materials in Mathematics Content Courses for Prospective Elementary School Teachers

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Purpose for Content Courses for Prospective Teachers

To develop deep understanding of mathematics that they are expected to teach.
Deep Understanding

Someone with a deep understanding can:
• consider a phenomenon or a problem from multiple perspective,
• represent the phenomenon or the problem in multiple ways,
• communicate his/her ideas thoroughly - logically and in details,
• etc.
Mathematics Teachers Are Expected to Teach

• Georgia Performance Standards
• GPS was heavily influenced by the 1989 Japanese Course of Study

“The new Georgia K–12 performance standards in mathematics support a strong, cohesive, and coherent curriculum that provides a clear path to higher mathematics and intelligent citizenship. They draw on the strengths of the Japanese school mathematics curriculum: coherence, leanness, and rigor.”
Mathematics Teachers Are Expected to Teach

- Georgia Performance Standards
- GPS was heavily influenced by the 1989 Japanese Course of Study
- No textbooks is developed for the GPS
- There are translated Japanese curriculum materials that are designed for the 1989 Japanese COS.
Japanese Curriculum Materials

- *Elementary School Teaching Guide for the Japanese Course of Study*

- *New Mathematics* by Hironaka & Sugiyama - the most widely used elementary school mathematics textbooks in Japan.
Teaching Guide

• Meaning of Addition/Subtraction
  – Combining
  – Increasing
  – Taking away
  – Comparing

• Number system
  – Rules of our number system
  – Merits of our number system
Teaching Guide

• Meaning of 0
  – Nothing
  – Place holder
  – A point of reference

• Multiplier & Multiplicand

• Equal Multiplication Principle
  – When the dividend and the divisor are both multiplied (or divided) by the same number, the quotient does not change (GPS M4N4d)
Teaching Guide

• Meaning of Fractions (2/3 as an example)
  – 2 parts of 3 equally divided whole
  – Measured quantity (such as 2/3 meters)
  – Two 1/3’s
  – Indicated division (GPS M5N4a)
  – Ratio
Japanese Textbooks

Structured problem solving

- Focus on a carefully selected problem
- Use only what we have studied together
- Students’ ideas are the center piece
- Instructor orchestrates classroom discourse

M*P1. Students will solve problems.

a. Build new mathematical knowledge through problem solving.

b. Solve problems that arise in mathematics and in other contexts.

c. Apply and adapt a variety of appropriate strategies to solve problems.

d. Monitor and reflect on the process of mathematical problem solving.
Japanese Textbooks

Representations
  - Mathematical representations
  - Manipulatives
  - Diagrams - thinking tools

M*P5. Students will represent mathematics in multiple ways.
   a. Create and use representations to organize, record, and communicate mathematical ideas.
   b. Select, apply, and translate among mathematical representations to solve problems.
   c. Use representations to model and interpret physical, social, and mathematical phenomena.
Example 1

- Meaning of Fractions (Decimals)
  \( \frac{a}{b} \) as a \( \frac{1}{b} \)’s

1. How long are 2, 3, and 4 pieces of \( \frac{1}{5} \) m?
   Please express these as fractions on a number line.
Example 1

• What is $\frac{2}{5} + \frac{4}{5}$?

There is $\frac{3}{5} \text{ l}$ of juice in a carton and $\frac{1}{5} \text{ l}$ in a bottle. How much juice is there altogether?

Let's think about how to calculate $\frac{3}{5} + \frac{1}{5}$.

How many $\frac{1}{5} \text{ l}$ are in each $\frac{3}{5} \text{ l}$ and in $\frac{1}{5} \text{ l}$?

$\frac{3}{5} \text{ l} \ldots \frac{1}{5} \text{ l}$
$\frac{1}{5} \text{ l} \ldots \frac{1}{5} \text{ l}$

$\frac{3}{5} + \frac{1}{5} = \square$  
Answer: $\square \text{ l}$
Example 1

- 0.8 - 0.3

There was 0.8ℓ of juice and the girl drank 0.3ℓ today. How much juice does she still have?

Let’s think about how to calculate 0.8 - 0.3!

How many 0.1’s are in 0.8 and in 0.3?

0.8 - 0.3 = [ ] ℓ

Answer: [ ] ℓ
Example 2

Tape Diagram

Stephen had some candies. He gave 6 to his friend, Joe. He now has 8 candies. How many candies did Stephen have at first?
Example 2

Tape Diagram

Stephen had some candies. He gave 6 to his friend, Joe. He now has 8 candies. How many candies did Stephen have at first?

Common Error (children’s)

8 - 6 = 2
Example 2

Tape Diagram

Stephen had some candies. He gave 6 to his friend, Joe. He now has 8 candies. How many candies did Stephen have at first?

What Stephen had

What was given away | What is left
Example 2

Tape Diagram

Stephen had some candies. He gave 6 to his friend, Joe. He now has 8 candies. How many candies did Stephen have at first?
Example 2

Tape Diagram

Casey read 7 more books than Jamie. If Casey has read 16 books, how many books did Jamie read?

\[ \text{Number of books Casey read} \]

\[ \text{Number of books Jamie read} \]

[Diagram showing the relationship between the number of books read by Casey and Jamie]
Example 2

Tape Diagram

Casey read 7 more books than Jamie. If Casey has read 16 books, how many books did Jamie read?

16

? 7
Practice Problems

• Jamie had 8 apples. Her mom gave her some more, and Jamie now has 12 apples. How many apples did she have at first?

• Kevin hit 5 fewer homeruns than David did. If Kevin hit 8 homeruns, how many homeruns did David hit?
Multiplication

- Cartesian Product (Combination)
- Area
- Rate
- Equal Sets
Multiplication

- Multiplicand: number in a group
- Multiplier: number of groups

Multiplicand \times Multiplier = Product
4 apples are in a bag. If there are 6 bags, how many apples are there altogether?
Division

Equal Sets

• Fair Sharing: given the number of groups, determine the number in a group.

• Measurement: given the number in a group, determine the number of groups.
Division

Fair Sharing

There are 24 apples. If we put them in 6 bags equally, how many apples will be in each bag?

24 apples

6 bags

0 1 2 3 4 5 6 bags

0 1 2 3 4 5 6
There are 24 apples. If we put 4 apples in a bag, how many bags will there be?
Practice Problems

• Ms. White has 340 connecting cubes for a math lesson. If 4 groups share the cubes equally, how many cubes will each group receive?

• With 1 gallon of gasoline, Mike’s car can travel 24 miles. How far can he go with 14 gallons of gasoline?
Double Number Line

1 \( m \) of copper wire weighs 120g. How much will 2.4\( m \) of the same wire weigh?
1 m of copper wire weighs 120g. How much will 2.4m of the same wire weigh?
4.2 m of copper wire weighs 380g. How much will 1 m of the same wire weigh?
0.8 m of copper wire weighs 160 g. How much will 1 m of the same wire weigh?
Multiplier & Product

0  ?  320  ?

0  0.7  1  2.4
Divisor & Quotient

0  160  ?  300

0  0.8  1  1.5
1

With 1dl of paint you can paint \( \frac{3}{5} \) m\(^2\) of boards. How many m\(^2\) can you paint with 2dl of paint?
Multiplication & Division of Fractions

\[ \frac{3}{5} \text{ means three } \frac{1}{5}'s \]

\[ \frac{3}{5} \times 3 = (3 \times 3) \frac{1}{5}'s, \text{ or } \frac{9}{5} \]

When you multiply a fraction by a whole number, keep the denominator the same and multiply the numerator by the whole number.
2. With 3 dl of paint you can paint \( \frac{4}{5} \) m² of boards. How many m² can you paint with 1 dl of paint?
Multiplication & Division of Fractions

Four $\frac{1}{5}$’s into 3 equal groups…

Find an equivalent fraction:

$\frac{(4 \times 3)}{(5 \times 3)}$, or $\frac{12}{5} \times \frac{1}{4}$’s into 3 equal groups.

Each group will have $\frac{4}{5} \times \frac{1}{3}$’s

⚠️ When you divide a fraction by a whole number, keep the numerator the same and multiply the denominator by the whole number.

\[
\frac{b}{a} \div c = \frac{b}{a \times c}
\]
With $\frac{2}{3}$ pint of paint, you can paint $\frac{9}{4}$ square feet of board. How much area can you paint with 1 pint of paint?
With $\frac{2}{3}$ pint of paint, you can paint $\frac{9}{4}$ square feet of board. How much area can you paint with 1 pint of paint?

How much can we paint with $\frac{1}{3}$ pint?

\[
\frac{9}{4} \div \frac{2}{3} = \left( \frac{9}{4} \div 2 \right) \times 3 = \frac{9}{4} \times 2 \times 3 = \frac{9 \times 3}{4 \times 2} = \frac{9}{4} \times \frac{3}{2}
\]
With \(\frac{2}{3}\) pint of paint, you can paint \(\frac{9}{4}\) square feet of board. How much area can you paint with 1 pint of paint?

\[
\frac{9}{4} ÷ \frac{2}{3} = \left(\frac{9}{4} \times 3\right) ÷ 2 = \frac{9 \times 3}{4} ÷ 2 = \frac{9 \times 3}{4 \times 2} = \frac{9}{4} \times \frac{3}{2}
\]

How much can you paint with 2 pints?
Mike’s car can travel 140 miles with 4.5 gallons of gasoline. How much can he travel with 6 gallons of gasoline?
Mike’s car can travel 140 miles with 4.5 gallons of gasoline. How much can he travel with 6 gallons of gasoline?
Mike’s car can travel 140 miles with 4.5 gallons of gasoline. How much can he travel with 6 gallons of gasoline?
Mike’s car can travel 140 miles with 4.5 gallons of gasoline. How much can he travel with 6 gallons of gasoline?

\[
140 \div 4.5 \times 6
\]
Mike’s car can travel 140 miles with 4.5 gallons of gasoline. How much can he travel with 6 gallons of gasoline?

\[
140 \div 3 \times 6
\]