Grade 9 Set A

Answer questions (1) through (4) below.

(1) Calculate \( \frac{2}{3} ÷ \frac{5}{7} \).

(2) Select the smallest number from A through E below.

\[
\begin{align*}
A & \quad \frac{1}{3} \\
B & \quad 0 \\
C & \quad -2 \\
D & \quad 4 \\
E & \quad -\frac{1}{2}
\end{align*}
\]

(3) Calculate \( 2 \times (-3)^2 \).

(4) Calculate \( 8 - 5 \times (-6) \).
Answers questions (1) through (4) below.

(1) Simplify \((2x + 7y) - 2(x - 3y)\).

(2) What is the value of the expression \(3a + 5b\) when \(a = 5\) and \(b = -4\)?

(3) There is a rectangle with the vertical dimension of \(a\) and the horizontal dimension of \(b\), as shown below.

What does the expression \(2(a + b)\) represent? Select the answer from A through E below.

A  Area of the rectangle.
B  Twice of the area of the rectangle.
C  Perimeter of the rectangle.
D  Twice the perimeter of the rectangle.
E  The length of the diagonals of the rectangle.

(4) Solve \(2x + 3y = 9\) in terms of \(y\).
Answer questions (1) through (4) below.

(1) We solved the linear equation \(7x = 5x + 6\) as shown below.

\[
\begin{align*}
7x & = 5x + 6 \quad \cdots \quad (1) \\
7x - 5x & = 6 \quad \cdots \quad (2) \\
2x & = 6 \\
x & = 3
\end{align*}
\]

To transform equation (1) to equation (2), we moved the term, \(5x\), from the right hand side to the left hand side. We can explain why this term can be moved across the equal sign using a property of equations.

From statements A through D, select the correct justification for moving the term \(5x\) across the equal sign.

A Since we can add the term \(5x\) to both sides of equation (1) while maintaining equality, we can move the term.

B Since we can subtract the term \(5x\) from both sides of equation (1) while maintaining equality, we can move the term.

C Since we can multiply both sides of equation (1) by 5 while maintaining equality, we can move the term.

D Since we can divide both sides of equation (1) by 5 while maintaining equality, we can move the term.

(2) Solve the linear equation \(4(x + 5) = 80\).

(3) We bought a total of 15 apples and oranges. Each apple costs 120 yen and each orange costs 70 yen. The total cost was 1600 yen.

Write a system of equations that can be used to find the number of apples, \(x\), and the number of oranges, \(y\).

You do not have to solve the system of equations.
(4) Solve the system of equations \[ \begin{align*}
5x + 7y &= 3 \\
2x + 3y &= 1
\end{align*} \]
Answer questions (1) and (2) below.

(1) Quadrilateral ABCD drawn on the grid below is symmetrical. From A through E, select the line of symmetry for quadrilateral ABCD.

A line AD
B line BC
C line EG
D line HF
E line AC

(2) When you are given $\angle XOY$, as shown in Figure 1, you can construct its bisector in steps (1), (2), and (3), shown in Figure 2.

Match each of steps (1), (2), and (3) with explanations A, B, and C below.

A Draw two circles with the same radius centered at A and B. Label their intersection P.
B Draw line OP.
C Draw a circle centered at O. Label the points where the circle intersects side OX and OY as A and B, respectively.
Answer questions (1) through (4) below.

(1) Answers questions ① and ② about the rectangular prism shown, right.

① Name one edge that is perpendicular to face EFGH.

② Name one edge that is skew to edge BF.

(2) A solid is created by rotating rectangle ABCD around line \( l \). The sketch of the resulting solid is included in Figures A through E. Name the correct sketch.
(3) One of figures A through E is the net of a cone that is shown on the right. Name the correct one.
(4) The picture below shows two container types: cylindrical and cone-shaped. Their circular bases are congruent to each other, and their heights are also equal. A cylindrical container is filled with water, and we will transfer the water into the cone-shaped containers.

One of the pictures A through E shows how the cone-shaped containers will be filled up after all the water from the cylindrical container is transferred. Select the correct picture.
Answer questions (1) through (3) below.

(1) Lines \( l \) and \( m \) below are parallel. Find the angle measure \( x \).

(2) In the figure below, points A, B, and P are on circle O. Moreover, \( \angle APB = 60^\circ \). Find the measure of angle measure \( y \).
(3) In quadrilateral $ABCD$, $AB\parallel DC$ and $AB=DC$. Therefore, we can conclude that quadrilateral $ABCD$ is a parallelogram.

Which of the following statements A ~ E represents the phrase "$AB\parallel DC$ and $AB=DC$."

A 2 pairs of opposite sides are parallel.
B 2 pairs of opposite sides are congruent.
C 2 pairs of opposite angles are congruent.
D The diagonals intersect at their mid-points.
E A pair of opposite sides are parallel and congruent.
A class proved the statement, "Opposite sides of a parallelogram are congruent," as shown below.

Draw the diagonal AC of parallelogram ABCD.

In $\triangle ABC$ and $\triangle CDA$, because alternate interior angles are congruent,

$\angle BAC = \angle DCA$ because $AB \parallel DC$, and ..... ①

$\angle BCA = \angle DAC$ because $AD \parallel BC$, and .....②

Moreover, $AC = CA$ since AC is in common. .......③

From statements ①, ②, and ③, we can say $\triangle ABC \cong \triangle CDA$ because two angles and the side in between them are congruent.

Therefore, $AB = CD$ and $BC = DA$.

Thus, the opposite sides of a parallelogram are congruent.

Statements A through D were made by students in the class. Which of the four statements is correct?

A  Even though we proved the statement as shown above, we still need to verify that opposite sides of a parallelogram are congruent by measuring them.

B  Even though we proved the statement as shown above, we still need to prove the statement again for a different parallelogram.

C  We can tell from the proof above that opposite sides of a parallelogram are congruent in any parallelogram.

D  From the proof above, we can tell that opposite sides of a trapezoid are also congruent.
Triangle ABC below is an isosceles triangle with $AB = AC$. Points D and E are on sides AB and AC, respectively, and $BD = CE$. The proof below shows that $CD = BE$.

In $\triangle DBC$ and $\triangle ECB$,
we are given that $BD = CE$. 

Because $\triangle ABC$ is an isosceles triangle, and the base angles of an isosceles triangles are congruent, 
$\angle DBC \cong \angle ECB$ 

Moreover, because $BC$ is in common
$BC = CB$

From statements ①, ②, and ③, we can conclude that $\triangle DBC \cong \triangle ECB$ because [ ].

Therefore, $CD = BE$. 
From statements A ~ E, select the appropriate congruent condition of triangles to fill in the blank in the proof.

A All 3 corresponding sides are congruent.

B 2 pairs of corresponding sides and the corresponding angles formed by the two sides are congruent.

C 2 pairs of corresponding angles and the corresponding sides between the angles are congruent.

D In right triangles, the hypotenuses and corresponding legs are congruent.

E In right triangles, the hypotenuses and a pair of corresponding acute angles are congruent.
Answer questions (1) and (2) below.

(1) From statements A ~ E, select a correct statement about the relationship between x and y when y is proportional to x.

A  When the value of x becomes 2, 3, ... times as much, the value of y becomes 2, 3, ... times as much.

B  When the value of x becomes 2, 3, ... times as much, the value of y becomes -2, -3, ... times as much.

C  When the value of x becomes 2, 3, ... times as much, the value of y becomes 4, 9, ... times as much.

D  When the value of x becomes 2, 3, ... times as much, the value of y becomes 1/2, 1/3, ... times as much.

E  When the value of x becomes 2, 3, ... times as much, the value of y becomes -1/2, -1/3, ... times as much.

(2) The graph below shows a proportional relationship. Write a mathematical expression for the value of y in terms of x.

![Graph](image_url)
The table below shows that $y$ is inversely proportional to $x$. Answer questions (1) and (2) below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>…</th>
<th>$-2$</th>
<th>$-1$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>…</td>
<td>$-6$</td>
<td>$-12$</td>
<td></td>
<td>12</td>
<td>6</td>
<td></td>
<td>□</td>
</tr>
</tbody>
</table>

(1) Find the number that goes into the □ in the table.

(2) One of the graphs A ~ E represents the relationship between $x$ and $y$ shown in the table. Select the correct one.
Answer questions (1) and (2) below.

(1) In one of the situations A ~ E below, the relationship between $x$ and $y$ is a linear function. Which one is it?

A  In a rectangle with the area of $60\, cm^2$, the horizontal dimension, $y\, cm$, when the vertical dimension is $x\, cm$

B  The total amount of water, $y\, liter$, after $x\, minutes$ when you pour water into a tank with 5 liter of water at the rate of 3 liter per minute

C  A person's weight, $y\, kg$, when his/her height is $x\, cm$

D  The length of ribbon, $y\, cm$, a person will receive when 6 m of ribbon is fairly shared by $x$ people

E  The temperature, $y\, °C$, at $x\, p.m.$

(2) One of the graphs below shows the graph of the linear function $y = -3x + 2$. Select the correct one.

A  

B  

C  

D  

E  
Manabu went to a park that is 700 m from his house. The graph below shows the relationship between the time and the distance from the house after he left home.

Answer questions (1) and (2) below.

(1) From the graph above, we can tell that Manabu traveled 100 m at a constant speed during the first 2 minutes. What was his speed in m/min.?

(2) What was his speed, in m/min., after the first 2 minutes until he reached the park?
In the graph below, line ① is the graph of $x + y = 5$ and line ② is the graph of $x - y = 1$. One of the points A ~ E has the coordinates which are the solution to the system of equations \[
\begin{align*}
    x + y &= 5 \\
    x - y &= 1
\end{align*}
\]
Which one is it? Please write A, B, C, D, or E.
Answer questions (1) and (2) below.

(1) When a die is rolled, the probability of rolling a 1 is 1/6. What can we say about rolling this die? Select one correct statement from the following statements A through E.

A  After you rolled the die five times without rolling a 1, you will certainly roll a 1 on the next roll.

B  If you roll the die six times, you will roll a 1 at least once.

C  If you roll the die six times, you will roll each of 1 through 6 once.

D  If you roll the die 30 times, you will roll a 1 exactly 5 times.

E  If you roll the die 3000 times, you will roll a 1 about 500 times.

(2) Teams A ~ E are playing volleyball games. If each team plays all the other teams once, how many games will be played all together?

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The table below summarizes the results of the survey conducted to find out how many books library representatives from 15 different classes read during the past month.

<table>
<thead>
<tr>
<th>Number of books read by library representatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Library Representative</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>Number of books read</td>
</tr>
</tbody>
</table>

Which one of the following statements A ~ E is correct about the mean of the number of books read by library representatives?

A  Most people, 5, read 4 books, so the mean is 4.

B  If the number of books read are arranged in a descending order, the middle, 8th, number is 3. Therefore, the mean is 3.

C  If you add the number of books read by the 15 representatives and then divide the total by 15, you will get 3.6. Therefore, the mean is 3.6.

D  The largest and the smallest number of books read are 8 and 2, respectively. Therefore, the mean is 5.
Kyoko and her family went to a family restaurant. All 5 of them decided to order meal sets.

**Save with Meal Sets!**

Select one item each from A, B, and C for only **1,050 yen!**

**A**
- Asparagus Salad
  - 150 Cal, sodium 2.8 g
- Clam Chowder
  - 200 Cal, sodium 2.1 g

**B**
- Supreme Pizza
  - 500 Cal, sodium 2.3 g
- Spaghetti
  - 400 Cal, sodium 3.5 g
- Omlette & Pilaf
  - 600 Cal, sodium 4.1 g

**C**
- Rainbow Sherbet
  - 200 Cal, sodium 0.2 g
- Pumpkin Pudding
  - 100 Cal, sodium 0.5 g
- Mango Sundae
  - 250 Cal, sodium 0.3 g

**Drinks**

+ 150 yen
  - Orange Juice
  - Oolong Tea

+ 200 yen
  - Coffee
  - Tea

(All prices include sales tax.)
Answer questions (1) through (3) below.

(1) Kyoko says, “I like Omelet & Pilaf, so I’m sure to order it.” If you pick Omelet & Pilaf from B, how many different set meals can you make by picking one item from A and one from C?

(2) Kyoko’s mother says, “I’m going to order an Asparagus Salad, but I have to watch calories and sodium. I want a set with the least amount of sodium with the total calories of 750 kcal or less.”

Select an item from B and an item from C to make the set that meets her request.

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Supreme Pizza</td>
<td>d. Rainbow Sherbet</td>
</tr>
<tr>
<td>b. Spaghetti</td>
<td>e. Pumpkin Pudding</td>
</tr>
<tr>
<td>c. Omelet &amp; Pilaf</td>
<td>f. Mango Sundae</td>
</tr>
</tbody>
</table>

(3) Because some of the five family members ordered drinks, the total bill was 5750 yen. Was there anyone who ordered a 200-yen drink? Answer A or B. Also explain why you chose your answer.

A  Yes  B  No
Taro is investigating the sums of 3 consecutive natural numbers.

When three numbers are 1, 2, and 3, 1+2+3=6
When three numbers are 2, 3, and 4, 2+3+4=9
When three numbers are 3, 4, and 5, 3+4+5=12

From these results, he conjectured that the sum of three consecutive natural numbers will be a multiple of 3. He justified his conjecture as shown below.

**Taro’s justification**

Let’s call the smallest of the three natural numbers \( n \).

Then, the three numbers can be written as \( n \), \( n+1 \), and \( n+2 \).

Therefore, their sum is

\[
\begin{align*}
 n + (n + 1) + (n + 2) &= n + n + 1 + n + 2 \\
 &= 3n + 3 \\
 &= 3(n + 1)
\end{align*}
\]

Since \( n + 1 \) is a natural number, \( 3(n + 1) \) is a multiple of 3.

Answer questions (1) and (2) below.

(1) In addition to the conclusion,

**the sum of 3 consecutive natural numbers is a multiple of 3**

there is another conclusion that can be drawn from the last expression in Taro’s justification, \( 3(n + 1) \). Select the correct conclusion that can be drawn from the following statements A through E.

- **A** The sum of 3 consecutive natural numbers is an odd number.
B  The sum of 3 consecutive natural numbers is an even number.

C  The sum of 3 consecutive natural numbers is 3 times as much as the smallest of the three numbers.

D  The sum of 3 consecutive natural numbers is 3 times as much as the middle number of the three.

E  The sum of 3 consecutive natural numbers is 3 times as much as the largest of the three numbers.

(2) Based on Taro's justification, we can conjecture that

the sum of 5 consecutive natural numbers is a multiple of 5.

Using Taro's justification as a reference, complete the justification of this conjecture below.

**Justification**

<table>
<thead>
<tr>
<th>Let the smallest of 5 natural numbers be $n$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Then, the 5 numbers may be expressed as $n, n + 1, n + 2, n + 3, \text{ and } n + 4$.</td>
</tr>
<tr>
<td>Therefore, the sum of the 5 consecutive numbers is expressed as $n + (n + 1) + (n + 2) + (n + 3) + (n + 4)$</td>
</tr>
<tr>
<td>$= n + n + 1 + n + 2 + n + 3 + n + 4$</td>
</tr>
</tbody>
</table>
In a soccer league, each of the 5 teams played all the other teams once. The results of the league play is summarized in the table below.

<table>
<thead>
<tr>
<th>Teams</th>
<th>Wins</th>
<th>Losses</th>
<th>Draws</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Q</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>R</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>S</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

In this league, the final standing is determined by the following rule.

**Rules for determining the final standing**

Each team will receive 3 points for a win, 0 point for a loss, and 1 point for a draw. The teams with more points will be ranked higher.

Answers questions (1) ~ (3) below.

1. Using the rule described above, determine the total points Team R earned during league play.
(2) Which team won the league? Select the answer from below.

A  Team P
B  Team Q
C  Team R
D  Team S
E  Team T

(3) According to the rule described above, a team's final standing is determined by the expression $3a + b$, where $a$ is the number of wins and $b$ is the number of draws. Mai says the following:

I thought if we changed the number of points a team earns for a win or a draw, without changing the rule for a loss, the standings may be different. So, I made a new expression. When I used the new expression, Teams Q and R earned the same number of points and they were tied for the first place.

Find the values of $a$ and $b$ so that the points for Teams Q and R are the same and they are tied for the first place. In addition, using the expression, justify that Teams Q and R are actually tied for first place.
As shown in the figure below, line \( l \) is the perpendicular bisector of segment AB, and \( l \) and AB intersect at point M. Point P is on line \( l \).

Someone tried to prove that \( PA = PB \) as shown below. However, there is an error in this proof.

**Proof:**

In \( \triangle PAM \) and \( \triangle PBM \), we are given that

\[
AM = BM \quad \ldots \ldots \text{①}
\]

and \( PA = PB \). \( \ldots \ldots \text{②} \)

Moreover, \( PM = PM \) (PM is in common) \( \ldots \ldots \text{③} \)

From statements ①, ②, and ③, we can conclude that

\( \triangle PAM \cong \triangle PBM \)

since all 3 pairs of corresponding sides are congruent.

Therefore, \( PA = PB \).

Answer questions (1) and (2) below.

(1) The error in the above proof is inside the section marked by [underline]. Underline (______) the erroneous statement inside [underline] on the answer sheet.
In $\triangle PAM$ and $\triangle PBM$, we are given that

\begin{align*}
AM &= BM \quad \ldots \ldots \text{(1)} \\
\text{and } PA &= PB. \quad \ldots \ldots \text{(2)}
\end{align*}

Moreover, $PM = PM$ (PM is in common) \ldots \ldots \text{(3)}

From statements (1), (2), and (3), we can conclude that

$\triangle PAM \cong \triangle PBM$

Since all 3 pairs of corresponding sides are congruent.

Therefore, $PA = PB$.

(2) Correct the proof below by re-writing the statements inside

In $\triangle PAM$ and $\triangle PBM$, we are given that

Therefore, $PA = PB$. 
In a science class, we conducted an experiment to determine how water temperature changes as it is heated. The graph below shows the changes in water temperature every two minutes from the time we began heating the water until 10 minutes after the experiment began.

Answer questions (1) ~ (3) below.

(1) What was the temperature of the water 10 minutes after we started heating it?

(2) As Yoko observed the graph, she thought that “if we consider the water temperature at $x$ minutes after we started heating the water is $y$ °C, we can say that $y$ is a linear function of $x$.” From what characteristic of the graph, can we conclude that “$y$ is a linear function of $x$”? Explain that characteristic.

(3) Hiroshi and Yoko were discussing how long it would take for the water to reach 80 °C. Following is the conversation between Hiroshi and Yoko.

Hiroshi: I have an idea.

Yoko: What is it? Please explain.
Hiroshi: We can just extend the graph so that we can read off the time it would take for the water to reach 80 °C.

Yoko: But if we extended the graph, it will go beyond the graph paper. We won't be able to read off the time.

What methods other than what was suggested by Hiroshi are possible to determine how long it would take for the water to reach 80 °C? You may not use additional graph paper to extend the graph nor change the scales on the axes.
Misaki went to the library to check out some books. The library is 1200 meters from her house. At a park on the way to the library, Misaki chatted with her friend for a little while before continuing on to the library. After she checked out the books, she directly came home along the same route without stopping at the park.

The graph below shows the relationship between the time after she left her house and the distance from the house.

Answer questions (1) ~ (3) below.

(1) What was Misaki doing during the time between points A and B on the graph?

(2) How long did Misaki stay in the library?
(3) From the graph above, we can tell whether Misaki was going faster from her home to the park or from the park to the library. When was she going faster? Answer with A or B, then explain why you chose your answer.

A  From the home to the park

B  From the park to the library