Leverage and Asymmetric Volatility: The Firm Level Evidence

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ABSTRACT

The relative statistical and economic significance of the leverage and feedback effects on firm level equity volatility is still an open issue in the finance literature. We use a dynamic panel vector autoregression framework to investigate both effects simultaneously for all firms in CRSP and COMPUSTAT from 1971 to 2005. An important feature of our methodology is that we allow leverage, volatility and risk premium to influence each other over time. We find a much larger leverage effect than reported in Christie (1982). Most importantly we find that a change in financial leverage has a prolonged effect on firm return volatility, i.e. the leverage effect is up to five times larger than a static model would predict.

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I. Introduction

Although there is a considerable literature on the modeling of equity volatility, the relative importance of the various theoretically identified determinants is still an open controversy. In particular, the importance of the leverage effect identified by Black (1976) is not yet fully understood. The leverage effect refers to the increase in stock volatility due to an increase financial in leverage. For example, a drop in equity price leads to an increase in financial leverage and an increase in equity volatility. This negative correlation between equity return and future volatility can sometimes also be explained by time varying risk premia. In this paper, we provide additional evidence on the importance of the leverage effect based on a large scale firm level study of equity volatility in an econometric model which allows for dynamic linkages between firm specific equity volatility, financial leverage, and time varying risk premia.

Our main finding is that financial leverage is an economically more significant determinant of equity volatilities than previous work has documented. Furthermore, we find that a change in financial leverage affects equity volatility for several periods and its impact on volatility accumulates over time. Our study suggests that past results may be due to not fully allowing for the endogenous nature of the relationship between leverage and volatility - the choice of leverage and business risk is a joint decision for a firm.

Christie (1982) documents that equity variance has a strong positive association with financial leverage and the negative elasticity of volatility with respect to the level of stock prices should be ascribed to financial leverage to a significant degree. This result is not without controversy. Figlewski and Wang (2000) use both returns and directly measured leverage to examine the effect of financial leverage as it applies to the individual stocks in the S&P100 (OEX) index, and to the index itself. They find a strong asymmetry associated with falling stock prices, but also numerous anomalies that call into question financial leverage changes as a viable explanation. They conclude that the “leverage effect” is rather a “down market effect” that may have little direct connection to firm capital structure.

An alternative explanation for the observed relationship between stock price levels and volatility is attributed to time varying risk premia. Following an increase in volatility priced by investors, required equity returns should increase thus leading to an immediate drop in the equity value. This story, which argues a causality opposed to the
financial leverage effect, has garnered support in the literature.\(^1\) Bekaert and Wu (2000) argue that the leverage explanation is in itself not sufficient and that the alternative explanation, often known as the volatility feedback effect, is supported by the data. Duffee (1995) studies the relationship between returns and volatility in a large sample of U.S. firms. He finds that the leverage effect is mainly due to the positive contemporaneous correlation between firm stock returns and firm stock return volatility; the correlation between returns and future volatility appears to be weak. In a dynamic general equilibrium model, Aydemir, Gallmeyer, and Hollyfield (2006) find that leverage contributes more to the dynamics of stock return volatility at the firm level.\(^2\)

Given the mixed results in the literature, we construct a model which allows for both channels between stock prices and volatility. To do so, we rely on a panel vector autoregressive framework to describe the dynamics of financial leverage, equity volatility, and risk premia. Our sample, an unbalanced panel, contains over 116,000 firm quarters during the period 1971-2005.\(^3\) *To the best of our knowledge this is the first study at the individual firm level to consider both volatility feedback and leverage.* In addition, we believe the scale of the study to be unprecedented, particularly in the context of a dynamic econometric model.

To establish a benchmark, we begin by estimating a bivariate panel vector autoregression that nests the Christie (1982) model. Unlike asymmetric GARCH type volatility modeling of leverage effect, we explicitly model the relationship between volatility and firm level leverage ratios. In doing so, we document a stronger relationship between equity volatility and the debt ratio than shown in Christie (1982). The coefficient estimates behave similarly across leverage quartiles, but they are economically more significant than in his study.

Our model allows for a bidirectional relationship between leverage and volatility, thus allowing for dynamic endogeneity between the two firm level choice variables, i.e. capital structure and business risk. We find that this is important in that the effect on volatility

\(^1\)Brown, Harlow, and Tinic (1988) show that stock price reactions to unfavorable news events tend to be larger than reactions to favorable events, and attribute their findings to volatility feedback. Poterba and Summers (1986), on the other hand argue against volatility feedback by pointing out that changes in volatility are too short-lived to have a major effect on stock prices.

\(^2\)For recent advances on market level volatility asymmetry see e.g. Bollerslev, Litvinova, and Tauchen (2006), Smith (2007), Hibbert, Daigler, and Dupoyet (2008), Bollerslev, Sizova, and Tauchen (2009).

\(^3\)The unbalanced structure of the dataset mitigates any potential sample selection biases.
of a change in leverage is long lasting and accumulates over time. Although the focus
of our study is on the relationship between leverage and volatility, we study to what
degree our results are dependent on the inclusion of time varying risk premia into the
system. We are comforted to find that our parameter estimates are robust to allowing
for an alternative explanation for the link between stock price levels and volatility.

To measure the accumulation of leverage and feedback effects, we use impulse re-
sponse functions. Consider for example the lowest leverage quartile: the immediate
effect of a one standard deviation shock to leverage is to increase the annualized volatil-
ity by about 2 per cent. However, the cumulative effect of the same shock to leverage
over the next 12 quarters exceeds 10 per cent annualized volatility. In the highest lever-
age quartile the cumulative effect can exceed 50 per cent annualized volatility. The
cumulative effect can easily multiply the direct impact of a leverage shock by 5 times.

Our setup allows us to study some of the implications of volatility feedback effect of
which we find some supporting evidence. Lagged volatility does have a positive effect
on the risk premium, but the effect does not accumulate over time in the way that the
effect of financial leverage does. In addition, we find a small but significant negative
contemporaneous correlation between the risk premium and the volatility.

In summary, we feel that our study provides strong evidence in support of the finan-
cial leverage effect on equity volatility, strengthening the conclusions of Christie (1982).
The accumulation of the leverage effect over time renders it at least up to five times
larger than previously thought.

The paper is organized as follows. Section II outlines the benchmark dynamic model
of leverage and volatility, addressing also the data used and the estimation methodology.
Section III considers the augmented model which allows for the volatility feedback effect.
In section IV, we discuss the core results of our study, extended in section V to consider
impulse response functions. Section VI concludes. Details about the panel vector
autoregression model are presented in appendix.

II. The dynamic panel data models

In the following section we introduce both, the bivariate and the trivariate panel vector
autoregression (PVAR) model. The bivariate model studies the bidirectional effect of
financial leverage and equity volatility over time. The trivariate model augments the
bivariate one with the risk premium, hence allows for both the leverage effect and the
A. A model for the leverage effect

This section introduces a bivariate PVAR model for the study on joint dynamics of leverage ratio and realized volatility. We use the fixed-effects specification in panel data. One advantage of fixed-effects panel models is the intercepts for each firm in the panel regression are allowed to be heterogeneous, which relax the usual constant-intercept assumption to allow for firm-level heterogeneity. Furthermore, the heterogeneous intercepts in a fixed-effects panel model are permitted to be correlated with the regressor(s), which may help to capture any effect from regressor(s) that cannot be explicitly modeled in a linear regression. Hence heterogeneous intercepts not only captures firm-level heterogeneity but also make the linear regression model robust to possible model misspecification. More details on advantages of using fixed-effects panel models can be found in Hsiao (2003).

In this paper, we further expand the advantage of panel modeling by incorporating vector autoregression (VAR) analysis to study joint dynamic relationships between the variables.

The estimation and inference in PVAR is first introduced in Holtz-Eakin et al. (1988), where the time series are assumed to be stationary and instrumental variable estimator is used. Binder et al. (2005) develop a quasi maximum likelihood estimator (QMLE) that allows for unit root processes in short PVAR. In our case, the panel time dimension is not short and therefore we adapt their method as follows.

Let \( w_{it} \) be a \( 2 \times 1 \) vector time series

\[
\begin{align*}
  w_{i,t} &= \begin{pmatrix} QR_{it} \\ \sigma_{it} \end{pmatrix} \\
  &\quad i = 1, \cdots, N \text{ and } t = 0, 1, \cdots, T_i,
\end{align*}
\]

where \( QR_{it} \) is the leverage ratio and \( \sigma_{it} \) is the realized volatility at time \( t \) for firm \( i \), and there are \( T_i + 1 \) observations for firm \( i \). We select volatility over variance for consistency with the theory laid out in Christie (1982).

Consider the following fixed-effects PVAR model

\[

w_{i,t} = a_i + \Phi_{2 \times 2} w_{i,t-1} + \varepsilon_{i,t},
\]

where

\[

\begin{align*}
  a_i &= a_{i,0} + \sum_{j=1}^{q} a_{i,j} w_{i,t-j} + \sum_{j=1}^{q} \pi_{i,j} \varepsilon_{i,t-j} \\
  \Phi_{2 \times 2} &= \begin{pmatrix} \phi_{1,1} & \phi_{1,2} \\ \phi_{2,1} & \phi_{2,2} \end{pmatrix},
\end{align*}
\]

\( q \) is the order of the VAR, \( a_i \) is the intercept, \( \phi_{ij} \) is the coefficient, and \( \varepsilon_{i,t} \) is the error term.
where
\[ a_i = \begin{pmatrix} a_{i1} \\ a_{i2} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}, \quad \text{and} \quad \varepsilon_{i,t} = \begin{pmatrix} \varepsilon_{it1} \\ \varepsilon_{it2} \end{pmatrix}. \]

The intercept vector \( a_i \) is allowed to be different for different firms and is the fixed-effects in our PVAR. A more explicit representation of the models is

\[
QR_{it} = a_{i2} + \phi_{11} QR_{i,t-1} + \phi_{12} \sigma_{i,t-1} + \varepsilon_{it1}, \quad (2)
\]

\[
\sigma_{it} = a_{i2} + \phi_{21} QR_{i,t-1} + \phi_{22} \sigma_{i,t-1} + \varepsilon_{it2}. \quad (3)
\]

The above model nests the linear regression model in Christie (1982). To see this, let us consider equation (3) which describes the dynamic relationship of volatility (\( \sigma_{it} \)) with leverage ratio (\( QR_{i,t-1} \)) and its own lag (\( \sigma_{i,t-1} \)). The coefficient \( \phi_{21} \) is the measure for leverage effect in Christie (1982), though no lags of \( \sigma_{it} \) appear in Christie’s regression model and the intercept is assumed to be constant for different firms in his regression. The VAR in (1) allows for interaction between leverage and volatility over the time and is a preferred approach to single-equation models whenever time series of both variables are available. Model (1) assumes that the parameters \( \Phi \) are homogenous. We address the issue of possible parameter heterogeneity in Section A.3.

We further assume

\[ \varepsilon_{i,t} \overset{i.i.d.}{\sim} (0, \Omega_{\varepsilon}), \]

where
\[
\Omega_{\varepsilon} = \begin{pmatrix} \sigma_{\varepsilon11} & \sigma_{\varepsilon12} \\ \sigma_{\varepsilon21} & \sigma_{\varepsilon22} \end{pmatrix}
\]

with \( \sigma_{\varepsilon12} = \sigma_{\varepsilon21} \).

The coefficient matrix \( \Phi \) in (1) can be estimated by taking the first difference of (1) to obtain

\[
\Delta w_{i,t} = \Phi \Delta w_{i,t-1} + \Delta \varepsilon_{i,t}, \quad (4)
\]

where \( \Delta w_{i,t} = w_{i,t} - w_{i,t-1} \) and \( \Delta \varepsilon_{i,t} = \varepsilon_{i,t} - \varepsilon_{i,t-1} \), then applying the quasi maximum likelihood method in Binder et al. (2005). The estimator is consistent and more details are given in Appendix.

Based on (1), we proceed in the next section to augment the model by incorporating risk premia in the dynamic system.
B. Accounting for Time varying risk premia

French, Schwert Stambaugh (1987), Campbell and Hentschel (1992), and more recently Bekaert and Wu (2000) point out that, in addition to the firms’ leverage, time varying risk premium could be an important reason for the negative correlation between equity return and future volatility. The varying risk premium hypothesis suggests that if volatility is priced, an anticipated increase in volatility raises the required return on equity, leading to an immediate stock price decline. This hypothesis applies directly to the market portfolio. For individual firms in a CAPM setup, the relevant measure of risk is instead the covariance of the stock with the market portfolio. The relationship in this case is indirect. In particular, the time-varying risk premium theory can contribute to explain firm-specific volatility asymmetry, through changes in the covariances with the market determined by changes in conditional volatility. The relationship between covariance risk and stock and market conditional volatility, and correlation can be better understood by rewriting the conditional covariance as

\[ \text{Cov}_{t-1}(r_{i,t}, r_{m,t}) = \rho_{im,t-1} \sigma_{i,t-1} \sigma_{m,t-1}, \]  

(5)

where \( \rho_{im,t} \) is the conditional correlation between market and firm \( i \), \( \sigma_{i,t} \) is the conditional volatility of the firm's stock and \( \sigma_{m,t} \) is market conditional volatility. The identity (5) serves to illustrate that the relationship between covariance and stock and market volatility depends on the sign of the correlation \( \rho_{im,t} \), and the magnitude of \( \rho_{im,t} \), \( \sigma_{i,t} \) and \( \sigma_{m,t} \). More precisely

\[ \frac{\partial \text{Cov}_{t-1}(r_{i,t}, r_{m,t})}{\partial \sigma_{i,t-1}} = \rho_{im,t-1} \sigma_{m,t-1} \]  

(6)

and hence, *ceteris paribus*, the individual stock risk premium should respond positively to increases in stock volatility only if stock return is positively correlated with the market return.

Equation (1) provides a system that can be thought as a dynamic version of Christie’s (1983) model. However, in addition to financial leverage, the most recent literature cited above shows that time varying risk premia are a viable explanation of volatility asymmetry. Our goal in what follows is to allow volatility to depend on a measure of the time-varying risk premium and verify whether the importance of the leverage variable \( QR_t \) in explaining firm volatility decreases.
Therefore, we assume that investors require returns consistently with the conditional CAPM. In particular, the conditional CAPM implies that

\[ E_{t-1}[r_{i,t}] = \frac{E_{t-1}[r_{m,t}]}{Var_{t-1}[r_{m,t}]} Cov_{t-1}[r_{i,t}, r_{m,t}], \]  

where \( r_{i,t} \) and \( r_{m,t} \) are the returns in excess of the T-Bill of asset \( i \) and the market, respectively, and \( E_t \) denotes the expectation operator conditional on the available information set. We take as an approximated measure of the quarterly required returns the realized expected returns computed using the daily data.

\[ \tau_{i,t-1} \equiv E_{t-1}[r_{i,t}] = \frac{\sum_{h=1}^{Q} r_{m,t-1,h}}{\sum_{h=1}^{Q} r_{m,t-1,h}^2} \sum_{h=1}^{Q} [r_{i,t-1,h} r_{m,t-1,h}], \]  

where \( Q \) is the number of days in each particular quarter, and \( h \) represents a day in the quarter. The expected returns variable \( \tau_{i,t} \) is essentially an ex-post measure of the required return based on the conditional CAMP. The advantage of this measure is that it does not need any estimation procedure per se as is based on realized variance and covariance (see e.g. Andersen Bollerslev (1998), Andersen et al. (2001, 2002, 2003)). Another desirable feature of this implementation of the conditional CAPM is that the variation in the risk premium can be driven by the time variability of any of its three components, namely variance, covariance and excess market return. In other words, equation (8) encompasses both the parametrization of the conditional CAMP with time varying beta and that with a time varying price of market risk. We study how well the conditional CAPM captures the changes in the risk premium by regressing the market returns on the expected returns computed from (8). We begin by setting up the following panel model with fixed-effects

\[ r_{i,t} = \alpha_i + \beta \tau_{i,t} + \varepsilon_{i,t}. \]  

The fixed-effects take into account firms heterogeneity and model firm level dependence on risk factors unaccounted by the conditional CAPM. Then we estimate (9) by taking the first differences of to eliminate the fixed-effects.
The constant $\alpha$ capture possible misspecification of the conditional CAPM and in particular the average residual cross sectional dependence of returns to risk factors unaccounted for by the conditional CAPM. As long as $\alpha$ is not statistically and economically significant, the changes in $\bar{\tau}$, $\Delta \bar{\tau}$ can be considered viable proxies for the purpose of the dynamic relationship among changes in volatility, leverage and expected returns in equation (10).

Under the null that $\tau_{i,t}$ is a suitable proxy of the risk premium, $\alpha$ should be 0 and $\beta$ should be 1. The estimate $\alpha$ is $-.002$ with a p-value of 13.4 per cent, thus statistically insignificant. At the same time, the coefficient $\beta$ is significantly different from 1 at 1.23, indicating that $\bar{\tau}_{i,t}$ is possibly a downward biased measure of the required returns. The $R^2$ of the regression is 13.22 per cent showing some non negligible predicting power. The misspecification of the CAPM is known in the literature. However, for tractability this is the only specification we can use to construct risk premia at the firm level. The considerations above and the fact that only first order differences in $\bar{\tau}_{i,t}$ are considered in the PVAR model (10) should mitigate concerns. These results in turn suggest that all the coefficients related to $\bar{\tau}_{i,t}$ in the panel VAR system (10) are possibly upwardly (in absolute terms) biased and thus may over estimate the importance of the feedback effect. We note that the above caveats on the specification of the risk premium do not affect the results of the first two equations in (10). However, the implications of the results in the third equation should be taken with caution.

With these caveats with regard to the definition of the risk premium variable, we then define an augmented VAR equation as follows. Let $w_{it}$ be a $3 \times 1$ vector time series which starts from time 0,

\[
w_{it} = \begin{pmatrix} Q R_{it} \\ \sigma_{it} \\ \bar{\tau}_{it} \end{pmatrix} \quad i = 1, \ldots, N \text{ and } t = 0, 1, \ldots, T_i,
\]
where \( QR_{it} \) and \( \sigma_{it} \), and \( \tau_{i,t} \), are the leverage ratio, the realized volatility, and stock risk premium, respectively.

The augmented model is

\[
\Delta QR_{i,t} = \phi_{11} \Delta QR_{i,t-1} + \phi_{12} \Delta \sigma_{i,t-1} + \phi_{13} \Delta \tau_{i,t-1} + \Delta \varepsilon_{it1} \\
\Delta \sigma_{it} = \phi_{21} \Delta QR_{i,t-1} + \phi_{22} \Delta \sigma_{i,t-1} + \phi_{23} \Delta \tau_{i,t-1} + \Delta \varepsilon_{it2} \\
\Delta \tau_{i,t} = \phi_{31} \Delta QR_{i,t-1} + \phi_{32} \Delta \sigma_{i,t-1} + \phi_{33} \Delta \tau_{i,t-1} + \Delta \varepsilon_{it3}
\]

with

\[
\Omega_\varepsilon = \begin{pmatrix}
\sigma_{\varepsilon11} & \sigma_{\varepsilon21} & \sigma_{\varepsilon31} \\
\sigma_{\varepsilon21} & \sigma_{\varepsilon22} & \sigma_{\varepsilon32} \\
\sigma_{\varepsilon31} & \sigma_{\varepsilon32} & \sigma_{\varepsilon33}
\end{pmatrix}
\]

In order to ensure a positive definite \( \hat{\Omega}_\varepsilon \) estimate, we reparameterize \( \Omega_\varepsilon \) as

\[
\Omega_\varepsilon = \begin{pmatrix}
\omega_{11}^2 & \omega_{11} \omega_{21} & \omega_{11} \omega_{31} \\
\omega_{11} \omega_{21} & \omega_{21}^2 + \omega_{22}^2 & \omega_{21} \omega_{31} + \omega_{22} \omega_{32} \\
\omega_{11} \omega_{31} & \omega_{21} \omega_{31} + \omega_{22} \omega_{32} & \omega_{31}^2 + \omega_{32}^2 + \omega_{33}^2
\end{pmatrix}
\]

The parameters vector becomes \( \theta = (\phi_{11}, \phi_{12}, \phi_{13}, \phi_{21}, \phi_{22}, \phi_{23}, \phi_{31}, \phi_{32}, \omega_{11}, \omega_{21}, \omega_{31}, \omega_{22}, \omega_{32}, \omega_{33})' \). In (10), both the feedback effect and leverage effect are taken into consideration. The parameters \( \phi_{21} \) and \( \phi_{32} \) capture the leverage effect and volatility feedback effect, respectively.

### III. Data description

The data are from the CRSP quarterly database and COMPUSTAT daily database merged using the identifiers CUSIP and CNUM. We use only one class of stock for each firm, the one for which the CUSIP’s last two digits are 10. The sample period starts the first quarter of 1971 and ends the fourth quarter of 2005. The quarterly volatilities \( \sigma_{it} \) are realized volatilities computed from the daily log returns. We use the Fama and French industry classification and drop the firms classified as financials, banks, and other. The debt is computed as the sum of total liabilities (data54) and preferred stock (data55). The value of equity is computed as the product of common shares outstanding (data61) and price at the end of the quarter (data14). The leverage variable \( QR_{i,t} \) is then defined...
as the ratio of debt over equity at time \( t \) for firm \( i \). Table I presents the descriptive statistics of the entire dataset and for the four quartiles. Quartiles are obtained based on the average \( QR_{i,t} \). Notice the high skewness of the leverage ratio due to some extremely high leverage.

Table I Panel 2 shows the same statistics for the dataset after eliminating the firms that have a leverage ratio \( QR \) above one hundred at any point in time. The reason we eliminate these outliers is twofold. Firstly, they have an undue influence on the estimation procedure. Secondly, from the inspection of the data we gather that the cause of the high leverage ratio is typically the extremely low equity, which in turn is suggestive of potential distress. Under such circumstances, the assumption that debt is going to be repaid at face value is no longer valid. Hence the book value of debt could not be used as a measure of debt’s value.

The resulting quarterly time series for each firm have in general different starting dates and lengths, which is important to avoid the sample selection bias. In other words, any attempt to obtain balanced dataset by choosing a subperiod and a subset of firms for which the data are available in that period may introduce sample selection bias. Using the entire universe of CRSP/COMPUSTAT firms mitigates this concern. Our results are obtained using the dataset without outliers.

Figure 1 shows the time series of the cross sectional averages of the leverage variable for each quarter, with one standard deviation bands. The variability of the cross sectional distribution of the first two sample moments appears related to the business cycles. We notice that when the average leverage level increases also the spread around the mean increases. The figure also shows how the cross sectional variance increases with the quartile.

Figure 2 similarly represents the realized volatility variable. We notice that when average volatility increases, also variance of realized volatility increases. We also note that this figure may not be as informative as the Figure 1 as firms are assigned to quartile by leverage, not realized volatility. Similarly, the plot of the expected return variable does not appear to be informative and is omitted to conserve space.
IV. Empirical results

Following Christie (1982) we partition the dataset on the basis of the average leverage. Each firm is assigned to a quartile increasing in leverage. We report estimates for the four quartiles and for the entire sample. This means that the firms in Q4 are those for which the average level of QR is the highest during their respective sample period.

We estimate the model by QMLE. We maximize the likelihood function in (12) using a hill-climbing algorithm that is robust to local optima using at least three arrays of randomly selected starting values. The robust standard errors are computed using numerical gradient and Hessian. One important property of the QMLE is that parameter estimates are robust to heteroskedasticity and serial correlation in the error term. As well, robust variance covariance estimate $V_{QMLE}$ is used for inference. Model diagnostic checking reveals that the first-order VAR is able to capture most of the dynamics of the original data series. Less parsimonious specifications, including a PVARMA(1,1) model, only marginally improves model fit and significantly increases the number of parameters without adding economic intuition.

Whitelaw (1994) and Brandt and Kang (2004) use VAR to study the dynamic relationship between volatility and expected returns at the market level. We are first to our knowledge to take into consideration the dynamic interrelation of volatility, leverage, and expected return at the firm level using panel data. Using VAR is important for our purpose as the dynamic setting can both shed some light on how the leverage effect and the feedback effect interrelate simultaneously, and how their relationship unfolds over time. On the one hand, the model allows the estimation of the covariance matrix of the error differences, which captures the contemporaneous correlation among the shocks to variables. On the other hand, the estimated coefficients illustrates whether these relationships cumulatively reinforce each other or rather tend to offset each other over time. The panel data methodology also takes care of the possible heterogeneity among firms and increases the estimation efficiency due to the large sample size. In the following subsections we examine these relationships.

A. Dynamic effects

As first step, we run a simple linear regression similar to Christie (1982). We cannot directly compare our results to Christie’s because the sample period and data are different and we have no way to retrieve his original data. Table (V) presents the results. The
coefficient $\beta_1$ presents a monotonically decreasing pattern as in Christie (1982). Nevertheless, the magnitude of the coefficients shows that the leverage effect is considerably larger than currently documented.

We argue that volatility and financial leverage affect each other in a dynamic fashion that the simple regression is unable to capture. The PVAR model in this paper is suited to measure this dynamic interaction between volatility and the leverage. Table VI shows the parameter estimated for the restricted model. This table is comparable to table 2 in Christie (1982) and to Table V. As the comparison of Tables VI and VII shows, results from the benchmark model (1) and the augmented model (10) are strikingly similar. We therefore discuss only the results in table VII. We examine the parameters related to leverage effect first, then those related to feedback, and the remaining ones last. The results in Table VII shows a large increase in the importance of firm leverage to explain individual stock volatility as compared with the results in Christie (1982). This change in magnitude of the leverage effect, clearly due to the different data set and sample period, is nevertheless an interesting finding.

A.1. The estimated leverage effect

The coefficient $\phi_{21}$ captures the relationship between volatility and lagged leverage. Its magnitude varies between 0.129 and 0.013. The sign is positive and the parameters are significant for all quartiles and for the entire sample. The magnitude is decreasing in leverage. This implies that firm volatility is less sensitive to changes in leverage when firms are more levered. In addition, parameter $\phi_{21}$ is monotonically declining across leverage quartiles as implied by Merton (1974) type of structural model of risky debt.\footnote{To see this, consider that in the Merton (1974) model the relationship between the volatility of equity $\sigma_S$ and the volatility of the levered firm $\sigma_V$ is such that $\sigma_S = \sigma_V \frac{V}{S} N[d_1]$, where $d_1 = \frac{\ln \left( \frac{V}{S} \right) + \frac{\sigma_V^2 T}{2}}{\sigma_S \sqrt{T}}$, and $V$ is the value of the levered firm, $K$ the face value of the debt, $r$ the risk free rate, $T$ the time to maturity of the debt, and $N[\cdot]$ denotes the cumulative normal probability. The same type of considerations can also explain the increasing pattern in leverage quartile of the coefficient $\phi_{12}$.} Our results confirm Christie’s that the rate at which the leverage affects volatility declines as financial leverage increases. However, the magnitude of the coefficient we estimate is much larger than that estimated by Christie: for the Q1 is about 19 times, for Q2 about 17 times, for Q3 is 19 times, for Q4 is about 7 times, and for the entire sample is about 5 times as large. In other words, our findings strengthen Christie’s conclusion that financial leverage is an important determinant of the volatility dynamics.
Brandt and Kang (2004) point out that one important aspect of using a dynamic model is that, depending on the sign of the coefficients, shocks to one variable may continue to affect all variables in the system to a different extent over a long period. In all the quartiles both $\phi_{12}$ and $\phi_{21}$ are positive and significant. This means that, *ceteris paribus*, a shock to either volatility or leverage will accumulate over time at a rate that depends on these two coefficients. This modeling feature sets our study aside from the extant literature. In fact, however small the effect of a change in leverage on volatility may seem by only looking at the first lag, when both the positive effects of lag volatility on leverage, and of lag leverage on volatility are taken into account, the cumulative effect of a shock to leverage on volatility becomes substantial. In the context of the augmented model, this result is illustrated through the plots of the cumulative impulse response functions in Figure 3. Figure 3 shows the cumulative effect of one orthogonalized standard deviation shock from $QR$ to $\sigma$. The horizontal axis measures the quarters and the vertical axis is expressed in percentage annualized volatility. The figure suggests that in all quartiles one standard deviation shock to $QR$ on $\sigma$ cumulates over the following 12 quarters to roughly six or seven times the effect observed after the first quarter. In addition, the same figure shows that whereas one standard deviation shock to $QR$ on $\sigma$ for firm in quartile one increases volatility of about 12 per cent over the next 12 quarters, for a firm in quartile four the increase is about 50 per cent annualized volatility over the next 12 quarters. This emphasizes the merit of using a VAR system to uncover the leverage effect over time, which is not discussed in the literature.

The coefficient $\phi_{31}$ captures the relationship between financial leverage and lag required returns. It is positive and significant, for the Q3, Q4, and for all sample. This suggests than an increase in leverage is followed by a higher required returns. The fact that this relationship is significant only for the firms with higher leverage ratio also suggests that the market requires a compensation only when a firm’s leverage increases from a relatively high level to an even higher level, and hence increases default risk.

**A.2. The estimated volatility feedback effect**

The coefficient $\phi_{32}$ captures the feedback effect at the one lag frequency. This positive sign is consistent with the feedback story as it shows that an increase in volatility is followed by an increase in expected returns. However, when this positive coefficient is considered jointly with the negative or mostly insignificant estimates of $\phi_{23}$ it appears that the lag structure of the VAR system dampens the effects of shocks to expected return.
and to volatility. This is the second important finding of our paper that highlights the importance of using a dynamic system. Furthermore, the fact that the slope coefficient in (9) is larger than one implies that \( \phi_{23} \) is upwardly biased, which further weakens the volatility feedback effect.

Figure 4 illustrates this point. It shows the cumulative effect of one orthogonal shock from \( \sigma \) to \( \tau \). The horizontal axis measures the quarters and the vertical axis is expressed in percentage annualized require excess return. The figure suggests that in all quartiles one standard deviation shock to from \( \sigma \) to \( \tau \) cumulates over the following 12 quarters. The cumulated effect on \( \tau \) from shocks to \( \sigma \) varies from 6 to 25 basis points over the next 12 quarters depending on the quartile. However, by comparing Figure 3 to Figure 4 it is apparent that the cumulated effect of such a shock is much smaller than for the case of a shock to the leverage variable \( QR \). In other words, the effect on \( \sigma \) from shocks to \( QR \) and the effect on \( \tau \) from shocks to \( \sigma \) are both significant at the one lag frequency. However, the dynamic structure of the system is such that over time the leverage effect cumulates more than the volatility feedback effect.

A.3. Other parameter estimates in the PVAR

The coefficient \( \phi_{11} \) captures the relationship between leverage and its own lag. Unsurprisingly, leverage is highly persistent, with highly significant values across the quartiles ranging between 0.83 for Q2 and 0.89 for the entire sample.

The coefficient \( \phi_{12} \) is positive for all quantiles. The sign is in agreement with the volatility feedback story. If firm level volatility increases, i.e., \( \Delta \sigma_{i,t-1} > 0 \), then according to volatility feedback story, \textit{ceteris paribus}, higher volatility raises the required rate of return on equity, which causes a decline in stock price. The decline in stock price increases leverage, thus \( \phi_{12} \) should be positive.

The dependence of the \( \phi_{12} \) and \( \phi_{21} \) on firm leverage implies slope heterogeneity in the PVAR model, i.e. \( \phi_{12} \) and \( \phi_{21} \) take different values for different firms with different leverage ratios. However, it can be shown that these coefficients can be seen as a random variables with \( \phi_i = \tilde{\phi} + e_i \) where \( e_i \) is a zero mean process. The QMLE procedure insures that \( \phi_{12} \) and \( \phi_{21} \) are consistent.\(^5\)

\(^5\)See Corollary 5.3 in White (1994)
Own lag of volatility, as measured by the coefficient $\phi_{22}$ has a positive and fairly large effect on current volatility. This effect is documented in the large literature on GARCH and increases as leverage increases. The magnitude of the coefficients should be interpreted while keeping into account the quarterly data frequency.

[Table VI about here]

The parameter $\phi_{13}$ captures the effect of lag required returns on financial leverage. This coefficient is negative, but insignificant for all the quartiles. However it is negative and highly significant for the entire sample. This negative relationship is consistent with the notion that when a firm experiences a decrease in the cost of capital, it also finds raising capital in the form of debt easier, and it may prefer the second alternative. The parameter $\phi_{31}$ is positive and significant for the entire data set. This is consistent with the fact already discussed above that when $QR$ increases, also $\sigma$ increases, and hence the required return must increase to compensate for the greater risk.

Finally, $\tau$ has a negative correlation with its own lag. The overall low statistical significance of the parameters in the third equation may be suggestive that model augmented to include the risk premium variable may add little information. This not however the case. The Wald test in table X rejects the hypothesis that the parameters $\phi_{13}, \phi_{23}, \phi_{31}, \phi_{32}, \phi_{33}$ are jointly zero, providing support for the augmented model. This finding is consistent with the recent literature highlighting the presence of the time varying risk premium to explain volatility asymmetry.

It is conceivable that changes in leverage determined by fluctuations of the market value of equity and changes in leverage determined by issuances or retirements of debt may affect equity volatility in a different manner. To shed some light on this issue we further augment (10) with a dummy variable which takes value one when debt increases.

$$
\Delta Q R_{i,t} = \phi_{11} \Delta Q R_{i,t-1} + \phi_{12} \Delta \sigma_{i,t-1} + \phi_{13} \Delta \tau_{i,t-1} + \Delta \varepsilon_{it1}
$$

$$
\Delta \sigma_{it} = \phi_{21} \Delta Q R_{i,t-1} + \phi_{22} \Delta \sigma_{i,t-1} + \phi_{23} \Delta \tau_{i,t-1} + \phi_{24} I[\Delta D_{i,t-1}>0] + \Delta \varepsilon_{it2}
$$

$$
\Delta \tau_{i,t} = \phi_{31} \Delta Q R_{i,t-1} + \phi_{32} \Delta \sigma_{i,t-1} + \phi_{33} \Delta \tau_{i,t-1} + \Delta \varepsilon_{it3}
$$

(11)

Table (VIII) shows the coefficients estimated for equation (11). The coefficient $\phi_{24}$ is always positive and significant and the remaining parameters are largely unchanged. In principle this results implies that, ceteris paribus, a change in leverage associated with an increase in debt increases volatility more than the same change in leverage when debt stay constant or decreases. However, the magnitude of these coefficients is economically
negligible and we therefore base the following discussion on (10).

**B. Contemporaneous correlation**

In addition to the intertemporal effects, the VAR system allows to make inferences about the contemporaneous correlation among financial leverage, volatility and estimated risk premia. Table IX shows the contemporaneous correlation between the shocks to changes in leverage, volatility, and required returns estimated from the trivariate PVAR system covariance matrix. The robust t-stats are computed using the delta method. In the context of our study, since we are using quarterly data, contemporaneous should be interpreted as “same quarter”, rather than “instantaneous”.

The correlations between shocks are all significant at any conventional level. The contemporaneous correlation between the shocks to changes in $QR$ and $\sigma$ is positive. This means that when volatility increases, *ceteris paribus*, either debt increases, or equity declines, or both. This result is supportive of a contemporaneous financial leverage effect. In fact, it characterizes one possible definition of the leverage effect. This correlation increases from 5.6 per cent for the firms in the first quartile to 13.2 per cent for the firms in the fourth quartile. It also is more pronounced for firms that are highly levered.

The contemporaneous correlation between the shocks to changes in $\tau$ and $\sigma$ is negative. It varies between $-7.1$ per cent for the first quartile to $-9.8$ per cent for the fourth quartile. This result is at odds with a contemporaneous feedback effect at the firm level. To observe the feedback effect this correlation should be positive. It is however consistent with the results in Brandt and Kang (2004) which study the same relationship at the market level. Note that this result is not in contrast with a positive relationship between risk and expected return. In fact in this framework not volatility, but covariance with the market return is the appropriate measure of risk at the firm level. We conjecture that this result may be due to the fact that expected returns move sluggishly with respect to firm volatility and leverage.

The contemporaneous correlation between required returns and leverage is negative. This may seem puzzling if one considers leverage as one measure of firm riskiness. However, even if there is not contemporaneous positive relationship between leverage and the remuneration required by investor to hold that risk, the sign turns positive on the first lag as shown by the coefficient $\phi_{31}$.

In summary, when we compare the evidence in favor of either leverage on feedback, we
find that the leverage effect is large and the dynamic system shows that its importance cumulates over a substantial number of lags. On the contrary, firstly there is no contemporaneous feedback effect, secondly the lag structure is such that the feedback effect observed at one lag frequency does not cumulate over time as much. These consideration are made more vividly clear by the inspection of the impulse response function discussed below.

[Table IX about here]

V. Impulse response function

In the following section we discuss the impulse response function from the PVAR system. In the impulse response functions, all the shocks are one standard deviation and are orthogonalized. The three different shocks are from $QR$, $\sigma$, and $\tau$, respectively. In each subplot each line represents the marginal effect of a shock to one of the three equations in the VAR system. For instance, the first subplot shows the marginal effect of a shock to $QR$ on $QR$, $\sigma$, and $\tau$ for firms in the first quartile. Notice that the lines in each plot are not directly comparable to each other due to different scaling an size of the shocks.

The first column of subplots shows that a shock from $QR$ has a positive and persistent effect on all the variables for all the quantiles. Similarly, a shock from realized volatility to the other variables has a positive effect on other variables. An increase in firm volatility has virtually no effect on a firm’s leverage when leverage is low. However, when leverage ratio increases as we move from the first quartile the fourth quartile, we observe leverage ratio will first increase then decrease. In particular, for firms with high leverage ratio in quartile four, the effect of an increase in volatility is so persistent that less than half of initial effect dies out after three years. We note that the $QR$ response to a shock from $\sigma$ is hump shaped for the firm with high $QR$. In other words, the largest effect of the shock occurs after three quarters.

For the third column of subplot we note that a shock from required returns has a negative effect on all the variable with the only exception of volatility in the first quartile. By inspecting the (red) circled line it appears that the effect of a shock for required returns on itself dissipates after one quarter. On the contrary its effect is quite persistent on $\sigma$. 
VI. Concluding remarks

The relative importance of leverage effect and the feedback effect are still debated in the literature. It appears that the two explanations are not mutually exclusive. Our main contribution is to shed some light on the dynamic effect of changes in the capitals structure on the volatility at the firm level in the tradition of Black (1976) and Christie (1982). As a side issue we address whether our results are robust to the inclusion of the volatility feedback as a determinant of asymmetric volatility.

We use a panel vector autoregression model to study the dynamic relationship among financial leverage, firm equity volatility, and time varying risk premium. We use a large unbalanced panel data set during the period 1971-2005. We believe that the scale of the study to be unprecedented, particularly in the context of a dynamic econometric model.

Our model allows for dynamic endogeneity among firm level financial leverage, equity volatility, and risk premium. The fixed-effects panel model controls for firms heterogeneity.

Firstly, we confirm that changes in financial leverage affect the volatility of stock returns at the firm level based on a large sample of firms and long data period. Our results strengthen the relationship between equity volatility and the debt ratio presented in Christie (1982).

Secondly and more importantly, we find that a dynamic set up is important to capture the cumulative leverage effect. A change in financial leverage has a prolonged effect on firm return volatility. The accumulation of the leverage effect over time renders it at least up to five times larger than previously thought. This finding is novel since the cumulative effects of changes in capital structure on volatility have not been examined in the literature. The impulse response functions illustrate that financial leverage is an economically more significant determinant of equity volatility than previous work has documented, and its effect accumulates over time.

Thirdly, we find some evidence of volatility feedback effect. However, the feedback effect is short lived and it does not accumulate over time in a fashion similar to that of financial leverage.

Finally, we find evidence of a positive contemporaneous relationship between equity volatility and financial leverage, but a negative correlation between required returns and volatility which is consistent with Brandt and Kang (2004) but inconsistent with the
volatility feedback effect story.

Our study suggests that past results may be due to not fully allowing for the endogenous nature of the relationship between capital structure and business risk.
VII. Appendix

We provide more details for QMLE of the coefficient matrix $\Phi$ in (1) and (4). Define

$$
\Delta \eta_i = \begin{pmatrix}
\Delta w_{i2} - \Phi \Delta w_{i1} \\
\vdots \\
\Delta w_{iT} - \Phi \Delta w_{i,T-1}
\end{pmatrix}_{2(T-1) \times 1}
$$

and the variance-covariance matrix of $\Delta \eta_i$ is given by

$$
\Sigma_{\Delta \eta} = \begin{pmatrix}
2\Omega_\varepsilon & -\Omega_\varepsilon & 0 \\
-\Omega_\varepsilon & 2\Omega_\varepsilon & -\Omega_\varepsilon \\
-\Omega_\varepsilon & -\Omega_\varepsilon & \ddots \\
0 & -\Omega_\varepsilon & 2\Omega_\varepsilon
\end{pmatrix}.
$$

Let the parameter vector be $\theta = (\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}, \sigma_{e11}, \sigma_{e12}, \sigma_{e22})'$. The log likelihood function for firm $i$ is given by

$$
l_i(\theta) = -\frac{2(T_i - 1)}{2} \log (2\pi) - \frac{1}{2} \log |\Sigma_{\Delta \eta}| - \frac{1}{2} \Delta \eta_i' \Sigma_{\Delta \eta}^{-1} \Delta \eta_i.
$$

The log likelihood function for all the firms is the summation of the individual likelihoods, $l(\theta) = \sum_i l_i(\theta)$, and the QMLE solves

$$
\max_{\theta} l(\theta)
$$

In order to ensure a positive definite $\hat{\Omega}_\varepsilon$, we reparameterize $\Omega_\varepsilon$ as

$$
\Omega_\varepsilon = \begin{pmatrix}
\omega_{11}^2 & \omega_{11}\omega_{12} \\
\omega_{11}\omega_{12} & \omega_{11}^2 + \omega_{22}^2
\end{pmatrix},
$$

and the parameter vector becomes $\theta = (\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}, \omega_{11}, \omega_{12}, \omega_{22})'$. The QMLE has asymptotic normal distribution

$$
\sqrt{N}(\hat{\theta} - \theta) \sim N(0, V_{\text{QMLE}}),
$$
where

\[ V_{QMLE} = H^{-1}GH^{-1} \quad (13) \]

with

\[
H = E \left[ -\frac{1}{N} \frac{\partial^2 l(\theta)}{\partial \theta \partial \theta'} \right],
\]

\[
G = E \left[ \frac{1}{N} \frac{\partial l(\theta)}{\partial \theta} \frac{\partial l(\theta)}{\partial \theta} \right].
\]

VIII. Tables and figures
Table I  
Descriptive statistics for the leverage variable QR.

### Panel 1: All Sample

<table>
<thead>
<tr>
<th></th>
<th>Q 1</th>
<th>Q 2</th>
<th>Q 3</th>
<th>Q 4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.184</td>
<td>0.540</td>
<td>1.315</td>
<td>6.984</td>
<td>2.315</td>
</tr>
<tr>
<td>Median</td>
<td>0.130</td>
<td>0.431</td>
<td>1.095</td>
<td>2.448</td>
<td>0.708</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.264</td>
<td>8.882</td>
<td>39.678</td>
<td>26377.000</td>
<td>26377.000</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.192</td>
<td>0.477</td>
<td>1.091</td>
<td>235.211</td>
<td>118.637</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>39.669</td>
<td>29.989</td>
<td>79.637</td>
<td>10342.310</td>
<td>40659.870</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1427190</td>
<td>1025705</td>
<td>7964801</td>
<td>1.34E+11</td>
<td>8.13E+12</td>
</tr>
<tr>
<td>Probability</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Observations</td>
<td>24365</td>
<td>31613</td>
<td>32061</td>
<td>30018</td>
<td>118057</td>
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</table>

### Panel 2: No Outliers

<table>
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<tr>
<th></th>
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<th>Q 2</th>
<th>Q 3</th>
<th>Q 4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.182</td>
<td>0.533</td>
<td>1.288</td>
<td>3.739</td>
<td>1.483</td>
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<tr>
<td>Median</td>
<td>0.129</td>
<td>0.425</td>
<td>1.066</td>
<td>2.395</td>
<td>0.698</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.264</td>
<td>8.882</td>
<td>39.678</td>
<td>98.387</td>
<td>98.387</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.190</td>
<td>0.473</td>
<td>1.089</td>
<td>5.284</td>
<td>3.068</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>38.352</td>
<td>30.871</td>
<td>81.692</td>
<td>74.022</td>
<td>195.074</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1311161</td>
<td>1082949</td>
<td>8279855</td>
<td>6.50E+06</td>
<td>1.82E+08</td>
</tr>
<tr>
<td>Probability</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Observations</td>
<td>24060</td>
<td>31367</td>
<td>31614</td>
<td>29820</td>
<td>116861</td>
</tr>
</tbody>
</table>
Table II
Descriptive statistics for the realized volatility. The annualized realized volatility is defined as $\sigma_{it} = \sqrt{\sum_{h=1}^{Q} r_{i,t-h}^2}$, for all the $Q$ days in a quarter.

<table>
<thead>
<tr>
<th>No Outliers</th>
<th>Q 1</th>
<th>Q 2</th>
<th>Q 3</th>
<th>Q 4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.594</td>
<td>0.574</td>
<td>0.521</td>
<td>0.488</td>
<td>0.542</td>
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<tr>
<td>Median</td>
<td>0.515</td>
<td>0.481</td>
<td>0.425</td>
<td>0.352</td>
<td>0.447</td>
</tr>
<tr>
<td>Maximum</td>
<td>10.360</td>
<td>7.399</td>
<td>6.728</td>
<td>7.893</td>
<td>10.360</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.375</td>
<td>0.380</td>
<td>0.409</td>
<td>0.450</td>
<td>0.408</td>
</tr>
<tr>
<td>Skewness</td>
<td>3.332</td>
<td>2.794</td>
<td>3.112</td>
<td>3.382</td>
<td>3.105</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>39.915</td>
<td>23.590</td>
<td>23.750</td>
<td>25.274</td>
<td>26.442</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1410678 594877.4 618184.5 6.73E+05 2.86E+06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability</td>
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<tr>
<td>Observations</td>
<td>24060</td>
<td>31367</td>
<td>31614</td>
<td>29820</td>
<td>116861</td>
</tr>
</tbody>
</table>

Table III
Descriptive statistics for the realized annualized expected return implied by the conditional CAPM. Both variance and covariance are ex-post realized measures and are computed from daily data. The expected return variable $\tau$ is defined as $\tau_{i,t} = \frac{\text{Cov}_{t-1}[r_{i,t}, r_{m,t}]}{\text{Var}_{t-1}(r_{m,t})} r_{m,t}$.

<table>
<thead>
<tr>
<th>No Outliers</th>
<th>Q 1</th>
<th>Q 2</th>
<th>Q 3</th>
<th>Q 4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.036</td>
<td>0.032</td>
<td>0.024</td>
<td>0.020</td>
<td>0.028</td>
</tr>
<tr>
<td>Median</td>
<td>0.016</td>
<td>0.013</td>
<td>0.011</td>
<td>0.009</td>
<td>0.012</td>
</tr>
<tr>
<td>Maximum</td>
<td>5.420</td>
<td>7.859</td>
<td>7.623</td>
<td>4.332</td>
<td>7.859</td>
</tr>
<tr>
<td>Minimum</td>
<td>-3.544</td>
<td>-3.057</td>
<td>-4.630</td>
<td>-4.697</td>
<td>-4.697</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.381</td>
<td>0.346</td>
<td>0.312</td>
<td>0.280</td>
<td>0.329</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.471</td>
<td>-0.109</td>
<td>-0.136</td>
<td>-0.365</td>
<td>-0.260</td>
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<tr>
<td>Jarque-Bera</td>
<td>73534.3 349308.4 654536.4 465905.9 1260513</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Observations</td>
<td>24060</td>
<td>31367</td>
<td>31614</td>
<td>29820</td>
<td>116861</td>
</tr>
</tbody>
</table>

24
Table IV
Unconditional correlation among the variables $QR$, $\sigma$, and $\overline{r}$.

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(QR,\sigma)$</td>
<td>0.083</td>
<td>0.14</td>
<td>0.133</td>
<td>0.28</td>
<td>0.114</td>
</tr>
<tr>
<td>$\rho(\overline{r},\sigma)$</td>
<td>-0.034</td>
<td>-0.04</td>
<td>-0.043</td>
<td>-0.02</td>
<td>-0.021</td>
</tr>
<tr>
<td>$\rho(QR,\overline{r})$</td>
<td>-0.070</td>
<td>-0.05</td>
<td>-0.033</td>
<td>-0.04</td>
<td>-0.046</td>
</tr>
</tbody>
</table>

Table V
Christie’s regressions. Cross sectional averages of the parameters estimates, and t-stats of the firm-wise regressions. The time subscript for $QR$ is consistent with Christie’s description of the variable as being constructed by dividing face value of debt at the end of the previous available data period by the value of the equity at the beginning of the period. The regression is augmented with the lag volatility to treat autocorrelation in volatility, which Christie treats before running the regression. The equation for each firm is then: $\sigma_t = \beta_0 + \beta_1 QR_{t-1} + \beta_2 \sigma_{t-1} + \varepsilon_t$

<table>
<thead>
<tr>
<th></th>
<th>Q 1</th>
<th>Q 2</th>
<th>Q 3</th>
<th>Q 4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.480</td>
<td>0.411</td>
<td>0.381</td>
<td>0.334</td>
<td>0.398</td>
</tr>
<tr>
<td>$t(\beta_0)$</td>
<td><strong>3.553</strong></td>
<td><strong>3.453</strong></td>
<td><strong>3.127</strong></td>
<td><strong>2.778</strong></td>
<td><strong>3.213</strong></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.192</td>
<td>0.069</td>
<td>0.039</td>
<td>0.019</td>
<td>0.074</td>
</tr>
<tr>
<td>$t(\beta_1)$</td>
<td>0.368</td>
<td>0.638</td>
<td>0.770</td>
<td>0.976</td>
<td>0.703</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.183</td>
<td>0.228</td>
<td>0.220</td>
<td>0.246</td>
<td>0.221</td>
</tr>
<tr>
<td>$t(\beta_2)$</td>
<td>1.460</td>
<td>1.850</td>
<td>1.775</td>
<td>1.800</td>
<td>1.734</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.123</td>
<td>0.169</td>
<td>0.188</td>
<td>0.229</td>
<td>0.180</td>
</tr>
<tr>
<td>$\beta_1/\beta_0$</td>
<td>0.401</td>
<td>0.168</td>
<td>0.101</td>
<td>0.056</td>
<td>0.186</td>
</tr>
</tbody>
</table>
Table VI
Bivariate PVAR system. Parameters estimates and robust t-stats. The parameters are the same as if the equations were expressed in levels.

\[ \Delta Q R_{i,t} = \phi_{11} \Delta Q R_{i,t-1} + \phi_{12} \Delta \sigma_{i,t-1} + \Delta \varepsilon_{i,t1} \]
\[ \Delta \sigma_{i,t} = \phi_{21} \Delta Q R_{i,t-1} + \phi_{22} \Delta \sigma_{i,t-1} + \Delta \varepsilon_{i,t2} \]

<table>
<thead>
<tr>
<th></th>
<th>Q 1 t-stats</th>
<th>Q 2 t-stats</th>
<th>Q 3 t-stats</th>
<th>Q 4 t-stats</th>
<th>All t-stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{1,1} )</td>
<td>0.8415</td>
<td>51.200</td>
<td>0.8343</td>
<td>84.754</td>
<td>0.8609</td>
</tr>
<tr>
<td>( \phi_{1,2} )</td>
<td>0.0111</td>
<td>2.441</td>
<td>0.0375</td>
<td>3.845</td>
<td>0.1407</td>
</tr>
<tr>
<td>( \phi_{2,1} )</td>
<td>0.1277</td>
<td>5.248</td>
<td>0.0684</td>
<td>8.772</td>
<td>0.0437</td>
</tr>
<tr>
<td>( \phi_{2,2} )</td>
<td>0.3557</td>
<td>9.123</td>
<td>0.3873</td>
<td>19.813</td>
<td>0.4139</td>
</tr>
<tr>
<td>( \omega_{1,1} )</td>
<td>0.0178</td>
<td>5.366</td>
<td>0.0246</td>
<td>6.399</td>
<td>0.0241</td>
</tr>
<tr>
<td>( \omega_{2,2} )</td>
<td>-0.2931</td>
<td>-27.609</td>
<td>0.2881</td>
<td>35.024</td>
<td>0.2885</td>
</tr>
</tbody>
</table>

Panel 2: No outliers

<table>
<thead>
<tr>
<th></th>
<th>Q 1 t-stats</th>
<th>Q 2 t-stats</th>
<th>Q 3 t-stats</th>
<th>Q 4 t-stats</th>
<th>All t-stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{1,1} )</td>
<td>0.8459</td>
<td>51.542</td>
<td>0.8337</td>
<td>82.744</td>
<td>0.8608</td>
</tr>
<tr>
<td>( \phi_{1,2} )</td>
<td>0.0086</td>
<td>2.419</td>
<td>0.0393</td>
<td>3.985</td>
<td>0.1381</td>
</tr>
<tr>
<td>( \phi_{2,1} )</td>
<td>0.1286</td>
<td>5.327</td>
<td>0.0697</td>
<td>8.861</td>
<td>0.0442</td>
</tr>
<tr>
<td>( \phi_{2,2} )</td>
<td>0.3534</td>
<td>8.923</td>
<td>0.3926</td>
<td>20.103</td>
<td>0.4127</td>
</tr>
<tr>
<td>( \omega_{1,1} )</td>
<td>0.1036</td>
<td>23.161</td>
<td>-0.2734</td>
<td>-29.898</td>
<td>-0.6352</td>
</tr>
<tr>
<td>( \omega_{2,1} )</td>
<td>0.0162</td>
<td>5.419</td>
<td>-0.0249</td>
<td>-6.355</td>
<td>-0.0247</td>
</tr>
<tr>
<td>( \omega_{2,2} )</td>
<td>0.2933</td>
<td>27.298</td>
<td>0.2872</td>
<td>34.829</td>
<td>-0.2915</td>
</tr>
</tbody>
</table>
Table VII

Trivariate PVAR system. Parameters estimates and robust t-stats. The table shows the estimate parameters for the following system. For the covariance matrix $\Omega$, we report the coefficients of the reparameterized matrix that insure semidefinite positiveness of the variance covariance estimate.

\[
\Delta Q R_{i,t} = \phi_{11} \Delta Q R_{i,t-1} + \phi_{12} \Delta \sigma_{i,t-1} + \phi_{13} \Delta \tilde{\sigma}_{i,t-1} + \Delta \varepsilon_{i1}
\]
\[
\Delta \sigma_{it} = \phi_{21} \Delta Q R_{i,t-1} + \phi_{22} \Delta \sigma_{i,t-1} + \phi_{23} \Delta \tilde{\sigma}_{i,t-1} + \Delta \varepsilon_{i2}
\]
\[
\Delta \tilde{\sigma}_{i,t} = \phi_{31} \Delta Q R_{i,t-1} + \phi_{32} \Delta \sigma_{i,t-1} + \phi_{33} \Delta \tilde{\sigma}_{i,t-1} + \Delta \varepsilon_{i3}
\]

Trivariate Panel VAR. No outliers.

<table>
<thead>
<tr>
<th></th>
<th>Q 1 t-stats</th>
<th>Q 2 t-stats</th>
<th>Q 3 t-stats</th>
<th>Q 4 t-stats</th>
<th>All t-stats</th>
<th>t-stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{1,1}$</td>
<td>0.8459</td>
<td>51.467</td>
<td>0.8329</td>
<td>80.647</td>
<td>0.8601</td>
<td>56.131</td>
</tr>
<tr>
<td>$\phi_{1,2}$</td>
<td>0.0083</td>
<td>2.355</td>
<td>0.0377</td>
<td>2.103</td>
<td>0.1363</td>
<td>4.773</td>
</tr>
<tr>
<td>$\phi_{1,3}$</td>
<td>-0.0085</td>
<td>-0.881</td>
<td>-0.1229</td>
<td>-0.151</td>
<td>-0.2469</td>
<td>-1.786</td>
</tr>
<tr>
<td>$\phi_{2,1}$</td>
<td>0.1291</td>
<td>4.060</td>
<td>0.0695</td>
<td>8.698</td>
<td>0.0442</td>
<td>11.729</td>
</tr>
<tr>
<td>$\phi_{2,2}$</td>
<td>0.3543</td>
<td>9.008</td>
<td>0.3916</td>
<td>20.072</td>
<td>0.4126</td>
<td>20.123</td>
</tr>
<tr>
<td>$\phi_{2,3}$</td>
<td>0.0411</td>
<td>1.639</td>
<td>-0.0571</td>
<td>-0.372</td>
<td>-0.0169</td>
<td>-7.544</td>
</tr>
<tr>
<td>$\phi_{3,1}$</td>
<td>-0.0002</td>
<td>-0.058</td>
<td>0.0006</td>
<td>0.274</td>
<td>0.0013</td>
<td>10.710</td>
</tr>
<tr>
<td>$\phi_{3,2}$</td>
<td>0.0038</td>
<td>1.465</td>
<td>0.0016</td>
<td>0.241</td>
<td>0.0004</td>
<td>1.197</td>
</tr>
<tr>
<td>$\phi_{3,3}$</td>
<td>-0.0370</td>
<td>-1.144</td>
<td>-0.0346</td>
<td>-1.553</td>
<td>-0.0302</td>
<td>-1.832</td>
</tr>
<tr>
<td>$\omega_{1,1}$</td>
<td>0.1036</td>
<td>23.178</td>
<td>0.2732</td>
<td>29.658</td>
<td>0.6349</td>
<td>18.133</td>
</tr>
<tr>
<td>$\omega_{2,1}$</td>
<td>0.0164</td>
<td>3.109</td>
<td>0.0248</td>
<td>3.478</td>
<td>0.0246</td>
<td>6.820</td>
</tr>
<tr>
<td>$\omega_{3,1}$</td>
<td>-0.0068</td>
<td>-15.078</td>
<td>-0.0076</td>
<td>-12.682</td>
<td>-0.0077</td>
<td>-28.542</td>
</tr>
<tr>
<td>$\omega_{2,2}$</td>
<td>0.2932</td>
<td>27.347</td>
<td>0.2871</td>
<td>34.897</td>
<td>0.2916</td>
<td>36.503</td>
</tr>
<tr>
<td>$\omega_{3,2}$</td>
<td>-0.0088</td>
<td>-18.398</td>
<td>-0.0056</td>
<td>-13.910</td>
<td>-0.0033</td>
<td>-16.374</td>
</tr>
<tr>
<td>$\omega_{3,3}$</td>
<td>-0.0955</td>
<td>-27.095</td>
<td>0.0865</td>
<td>33.287</td>
<td>-0.0781</td>
<td>-31.943</td>
</tr>
</tbody>
</table>
Table VIII

Trivariate PVAR system with debt dummy. The indicator function takes value one if debt increases. Parameters estimates and robust t-stats. The table shows the estimate parameters for the following system. For the matrix \( \Omega_{i,t} \), we report the coefficients of the reparameterized covariance matrix that insure semidefinite positiveness of the estimate.

\[
\begin{align*}
\Delta Q_{i,t} &= \phi_{11} \Delta Q_{i,t-1} + \phi_{12} \Delta \sigma_{i,t-1} + \phi_{13} \Delta \tau_{i,t-1} + \Delta \varepsilon_{i,t} \\
\Delta \sigma_{i,t} &= \phi_{21} \Delta Q_{i,t-1} + \phi_{22} \Delta \sigma_{i,t-1} + \phi_{23} \Delta \tau_{i,t-1} + \phi_{24} \mathbb{I}[\Delta D_{i,t-1} > 0] + \Delta \varepsilon_{i,t} \\
\Delta \tau_{i,t} &= \phi_{31} \Delta Q_{i,t-1} + \phi_{32} \Delta \sigma_{i,t-1} + \phi_{33} \Delta \tau_{i,t-1} + \Delta \varepsilon_{i,t}
\end{align*}
\]

Table IX

Contemporaneous correlation of the shocks among the three variables \( QR, \sigma, \) and \( \tau \). The robust t-stats are computed using the delta method.

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>t-stats</th>
<th>Q2</th>
<th>t-stats</th>
<th>Q3</th>
<th>t-stats</th>
<th>Q4</th>
<th>t-stats</th>
<th>All</th>
<th>t-stats</th>
<th>All</th>
<th>t-stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(QR, \sigma) )</td>
<td>0.056</td>
<td>554.74</td>
<td>0.086</td>
<td>487.98</td>
<td>0.084</td>
<td>605.24</td>
<td>0.132</td>
<td>409.50</td>
<td>0.082</td>
<td>981.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho(\tau, \sigma) )</td>
<td>-0.071</td>
<td>-788.15</td>
<td>-0.088</td>
<td>-812.20</td>
<td>-0.098</td>
<td>-695.05</td>
<td>-0.087</td>
<td>-508.72</td>
<td>-0.053</td>
<td>-1641.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho(QR, \tau) )</td>
<td>-0.096</td>
<td>-733.27</td>
<td>-0.071</td>
<td>-251.86</td>
<td>-0.050</td>
<td>-195.49</td>
<td>-0.066</td>
<td>-188.75</td>
<td>-0.070</td>
<td>-1094.12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Wald Test of linear restriction. The table shows the test that $H_0$: \( \phi_{13}, \phi_{23}, \phi_{31}, \phi_{32}, \phi_{33} \) are jointly zero. The strong rejection of the null for all the quartiles highlights the importance of the \( \tau_t \) variable to explain the dynamics of the leverage and the volatility.

<table>
<thead>
<tr>
<th>Quartile</th>
<th>Q 1</th>
<th>Q 2</th>
<th>Q 3</th>
<th>Q4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wald statistics</td>
<td>342.155</td>
<td>295.482</td>
<td>41.270</td>
<td>326.315</td>
<td>908.035</td>
</tr>
<tr>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
References


Figure 1. Cross Sectional Average QR, with +/- One Standard Deviation Bands.
Figure 2. Cross Sectional Average Realized Volatility, with +/- One Standard Deviation Bands.
Figure 3. Cumulative Effects of an Orthogonal Shock (1 s.d. of QR) to QR on $\sigma$.
Figure 4. Cumulative Effects of an Orthogonal Shock (1 s.d. of $\sigma$) to $\sigma$ on $r_{bar}$
Figure 5 - Impulse response: Response to one std shock. Legend: * blue = QR; + green = $\sigma$; o red = E[r]