Finite Sample Properties of FGLS Estimator for Random-Effects Model under Non-normality

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Abstract
This paper considers the finite sample properties of the feasible generalized least square (FGLS) estimator for the random-effects model with non-normal errors. By using the asymptotic expansion, we study the effects of skewness and excess kurtosis on the bias and Mean Square Error (MSE) of the estimator. The numerical evaluation of our results is also presented.

Key Words: finite sample, non-normality, panel data, random-effects

JEL Classification: C1, C4

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1 Introduction

In random-effects (error component) models when variances of the individual-specific effect and error term are unknown, feasible generalized least square (FGLS) is the standard way for estimation (Baltagi, 2001). For large sample size, FGLS has the same asymptotic efficiency as the GLS estimator when variances are known (Fuller and Battese, 1974). However, we deal with data sets of small and moderately large sample size in many situations and the disturbances are typically believed to be non-normally distributed.

Maddala and Mount (1973) provided a simulation study on the efficiency of slope estimators for a static one-way error component panel data model. They considered both normal and non-normal errors in simulations, where their non-normal errors are from lognormal distribution. It is found that maximum likelihood estimator performs as well as other types of FGLS estimators under both normal and lognormal errors in small samples and all estimators give equally well results. Baltagi (1981) investigated thoroughly various estimation and testing procedures in a static two-way error component model and extended many estimation results in one-way models to two-way models. Taylor (1980) examined the exact analytical small sample efficiency of FGLS estimator compared to between groups estimator and within groups estimator under the assumption of normality.
Despite of previous studies, there has been no analytical result on how non-normality affects the statistical properties of FGLS estimator in static panel data model when sample size is finite. Further, we note that the exact analytical result for the non-normal case is difficult to obtain and it needs the specification of the form of the non-normal distribution. This paper gives the large-n (fixed T) approximate analytical result of finite sample behavior of FGLS with non-normal disturbances. We derive the approximate bias, up to $O(1/n)$, and the mean square error (MSE), up to $O(1/n^2)$, of the FGLS estimator in a static regression model under the assumption that the first four moments of the errors are finite. For the case of dynamic panel, the finite sample properties has been studied in several papers through simulation, for example, Nerlove (1967, 1971), Arellano and Bond (1991), and Kiviet(1995), and they are not directly related to the static case studied in this paper.

The paper is organized as follows. Section 2 gives the main results. Section 3 are detailed proofs. Some numerical results are given and discussed in Section 4. Section 5 provides the conclusion.
2 Main Results

Let us consider the following random effect model,

\[ y_{it} = x_{it}\beta + w_{it}, \]  \hspace{1cm} (2.1)

\[ w_{it} = \alpha_i + u_{it}, \quad i = 1, \ldots, n, t = 1, \ldots, T, \]

where \( y_{it} \) is the dependent variable, \( x_{it} \) is an \( 1 \times k \) vector of exogenous variables, \( \beta \) is a \( k \times 1 \) coefficient vector and the error \( w_{it} \) consists of a time-invariant random component, \( \alpha_i \), and a random component \( u_{it} \). We can also write the above model in a vector form as

\[ y = \begin{bmatrix} Xz + w, \end{bmatrix} \]  \hspace{1cm} (2.2)

\[ w = \begin{bmatrix} D\alpha + u, \end{bmatrix} \]

\[ D = \begin{bmatrix} I_n \otimes \nu_T, \end{bmatrix} \]

where \( y \) is \( nT \times 1 \), \( X \) is \( nT \times k \), \( w \) is \( nT \times 1 \), \( \alpha \) is \( n \times 1 \), \( I_n \) is an identity matrix of dimension \( n \), and \( \nu_T \) is \( T \times 1 \) with all elements equal to one.
We assume both \( \alpha_i \) and \( u_{it} \) are i.i.d. and mutually independent and

\[
E\alpha_i = 0, \quad E\alpha_i^2 = \sigma_\alpha^2, \quad E\alpha_i^3 = \sigma_\alpha^3\gamma_{1\alpha}, \quad E\alpha_i^4 = \sigma_\alpha^4(\gamma_{2\alpha} + 3), \tag{2.3}
\]

\[
E\alpha_i\alpha_j = \begin{cases} 
\sigma_\alpha^2 & \text{if } i = j, \\
0 & \text{if } i \neq j,
\end{cases}
\]

\[
Eu_{it} = 0, \quad Eu_{it}^2 = \sigma_u^2, \quad Eu_{it}^3 = \sigma_u^3\gamma_{1u}, \quad Eu_{it}^4 = \sigma_u^4(\gamma_{2u} + 3),
\]

\[
Eu_{it}u_{js} = \begin{cases} 
\sigma_u^2 & \text{if } i = j, t = s, \\
0 & \text{otherwise},
\end{cases}
\]

\[
E\alpha_ix_{it} = Eu_{js}x_{it} = 0, \quad i, j = 1, \ldots, n \text{ and } s, t = 1, \ldots, T,
\]

where \( \gamma_{1\alpha}, \gamma_{1u} \) and \( \gamma_{2\alpha}, \gamma_{2u} \) are Pearson’s measures of skewness and kurtosis of the distribution.

The variance-covariance matrix of \( w \) can be written as

\[
Eww' = \sigma_u^2(Q + \lambda^{-1}Q) \tag{2.4}
\]

\[
= \sigma_u^2\Omega^{-1},
\]

where \( Q = I_{nT} - \hat{Q}, \quad \hat{Q} = DD'/T, \lambda = \sigma_u^2/\sigma_\eta^2 \) and \( \sigma_\eta^2 = \sigma_u^2 + T\sigma_\alpha^2, \) \( 0 < \lambda \leq 1.\)
Obviously, we have the following properties of $Q$ and $\tilde{Q}$:

\[ Q^2 = Q, \ \tilde{Q}^2 = \tilde{Q}, Q\tilde{Q} = 0, \text{ and } \Omega = Q + \lambda \tilde{Q} = I_n - (1 - \lambda)\tilde{Q}. \] (2.5)

The generalized least square (GLS) estimator of $\beta$ when the variances of $u_{it}$ and $\alpha_i$ are known is given by

\[ \hat{\beta}_{GLS} = (X'\Omega X)^{-1}X'y. \] (2.6)

When the variances of $u_{it}$ and $\alpha_i$ are unknown, then feasible GLS estimator is used by replacing $\Omega$ with its estimator, $\hat{\Omega}$,

\[ \hat{\beta}_{FGLS} = (X'\hat{\Omega} X)^{-1}X'\hat{\Omega}y, \] (2.7)

where

\[ \hat{\Omega} = Q + \hat{\lambda} \tilde{Q}, \] (2.8)

\[ \hat{\sigma}_u^2 = \frac{u'(Q - QX(X'QX)^{-1}X'Q)u}{n(T - 1) - k}, \] (2.9)

\[ \hat{\sigma}_\eta^2 = \frac{w'(\tilde{Q} - \tilde{Q}X(X'\tilde{Q}X)^{-1}X'\tilde{Q})w}{n - k}, \] (2.10)

\[ \hat{\lambda} = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_\eta^2}. \] (2.11)
By expanding the terms in (2.8)-(2.11) and plugging them into (2.7), we obtain the analytical expression of the second-order bias and mean square error for $\hat{\beta}_{FGLS}$. The detailed proofs are given in Section 3 and we give the main result in the following theorem.

**Theorem 2.1.** Under assumption (2.3) the large-sample asymptotic approximations for the bias vector $E(\hat{\beta}_{FGLS} - \beta)$ up to $O(n^{-1})$ and mean square error matrix $E \left( (\hat{\beta}_{FGLS} - \beta)(\hat{\beta}_{FGLS} - \beta)' \right)$ up to $O(n^{-2})$ are given by

$$
\text{Bias} = \frac{\lambda(1 - \lambda)}{n^2} \left( \sigma_u \gamma_{1u} - \sigma_{\alpha} \gamma_{1\alpha} \right) (A^{-1} - \lambda A^{-1} BA^{-1}) X' \iota_n T,
$$

$$
MSE = \sigma^2_u (X' \Omega X)^{-1} + \frac{\lambda \sigma^2_u}{n^2} \left[ \frac{2T}{T - 1} - \frac{\gamma_{2u}}{T} (1 - \lambda)^2 - \gamma_{2\alpha} (1 - \lambda)^2 \right] \Delta + C + \frac{C'}{n},
$$

where $\iota_n T$ is an $nT \times 1$ vector of ones, $A = \frac{1}{n} X' \Omega X$, $B = \frac{1}{n} X' \bar{Q} X$, $\Delta = A^{-1} (B - \lambda B A^{-1} B) A^{-1}$, and

$$
C = \frac{\lambda \sigma^2_u}{n} A^{-1} X' \Omega \sqrt{n} \left( I \odot \bar{Q} X \left( X' \bar{Q} X \right)^{-1} X' \bar{Q} \right) 
$$

$$
- \frac{\gamma_{2u}}{T - 1} \left( I \odot Q X \left( X' Q X \right)^{-1} X' Q \right)
$$

$$
+ \frac{\gamma_{2\alpha} (1 - \lambda)^2}{\lambda T^2} D \left( I \odot D X \left( X' \bar{Q} X \right)^{-1} X' D \right) D' P_1 A^{-1},
$$

in which $P_1 = (X' \bar{Q} - BA^{-1} X' \Omega) / \sqrt{n}$.

The proof of Theorem 2.1 is given in Section 3. When errors are normally
distributed, $\gamma_{1\alpha} = \gamma_{2\alpha} = \gamma_{1u} = \gamma_{2u} = 0$ and we get

**Corollary 2.1.** Under assumption (2.3), when the errors are normally distributed, the large-sample asymptotic approximations for the bias vector $E(\hat{\beta}_{FGLS} - \beta)$ up to $O(n^{-1})$ and mean square error matrix $E \left( (\hat{\beta}_{FGLS} - \beta)(\hat{\beta}_{FGLS} - \beta)' \right)$ up to $O(n^{-2})$ are given by

$$
\text{Bias} = 0,
$$

$$
\text{MSE} = \sigma_u^2 (X'\Omega X)^{-1} + \frac{2\lambda \sigma_u^2 T}{n^2(T-1)} \Delta.
$$

If the non-normality comes from $\alpha_i$, not from $u_{it}$, then $\gamma_{1u} = \gamma_{2u} = 0$ and we have

**Corollary 2.2.** Under assumption (2.3), when only $\alpha_i$ is non-normally distributed, the large-sample asymptotic approximations for the bias vector $E(\hat{\beta}_{FGLS} - \beta)$ up to $O(n^{-1})$ and mean square error matrix $E \left( (\hat{\beta}_{FGLS} - \beta)(\hat{\beta}_{FGLS} - \beta)' \right)$ up to $O(n^{-2})$ are given by

$$
\text{Bias} = -\frac{\lambda(1 - \lambda)\sigma_u\gamma_{1\alpha}}{n^2} (A^{-1} - \lambda A^{-1}BA^{-1})X'\ell_{nt},
$$

$$
\text{MSE} = \sigma_u^2 (X'\Omega X)^{-1} + \frac{2\lambda \sigma_u^2 T}{n^2(T-1)} \gamma_{2\alpha} (1 - \lambda)^2 \Delta + \frac{F}{n} + \frac{F'}{n},
$$

where $F = \frac{\lambda \sigma_u^2}{n} A^{-1} X'\Omega \sqrt{n} \left[ \frac{\gamma_{2\alpha}(1 - \lambda)^2}{\lambda T^2} D \left( I \otimes DX(X'\bar{Q}X)^{-1} X'D \right) D' \right] P_1'^{-1} A^{-1}$.  

Similarly, if the non-normality comes only from $u_{it}$, then $\gamma_{1\alpha} = \gamma_{2\alpha} = 0$ and we have

**Corollary 2.3.** Under assumption (2.3), when only $u_{it}$ is non-normally distrib-
uted, the large-sample asymptotic approximations for the bias vector $E(\hat{\beta}_{FGLS} - \beta)$ up to $O(n^{-1})$ and mean square error matrix $E((\hat{\beta}_{FGLS} - \beta)(\hat{\beta}_{FGLS} - \beta)'$ up to $O(n^{-2})$ are given by

$$
\text{Bias} = \frac{\lambda (1 - \lambda) \sigma_u \gamma_{1u}}{n^2 T} (A^{-1} - \lambda A^{-1} B A^{-1}) X' l_{nT},
$$

$$
\text{MSE} = \sigma_u^2 (X' \Omega X)^{-1} + \frac{\lambda \sigma_u^2}{n^2 T} \left[ - \frac{\gamma_{2u}}{T} (1 - \lambda)^2 \Delta + \frac{G}{n} + \frac{G'}{n} \right],
$$

where

$$
G = \frac{\lambda \sigma_u^2}{n} A^{-1} \frac{X' \Omega}{\sqrt{n}} \left[ \lambda \gamma_{2u} \left( I \otimes \tilde{Q} X \left( X' \tilde{Q} X \right)^{-1} X' \tilde{Q} \right) ight. \\
- \frac{\gamma_{2u}}{T - 1} \left( I \otimes Q X \left( X' Q X \right)^{-1} X' \tilde{Q} \right) \left] P_1 A^{-1} \right.
$$

We note that the asymptotic MSE of $\hat{\beta}_{FGLS}$ is given by $\sigma_u^2 (X' \Omega X)^{-1}$. The following remarks follow from the results in Theorem 2.1 and Corollary 2.1.

**Remark 2.1.** The Bias depends only on skewness coefficient. Bias is zero if $\lambda = 1$ or $\lambda = 0$, where $\lambda = 1$ implies $\sigma_\alpha^2 = 0$ and $\lambda = 0$ implies $\sigma_u^2 = 0$. Also note that for symmetric distributions, $\gamma_{1\alpha} = \gamma_{1u} = 0$, or for distributions satisfying $\gamma_{1u} / \gamma_{1\alpha} = T \sigma_\alpha / \sigma_u$, Bias is zero. Consider the term

$$
A^{-1} - \lambda A^{-1} B A^{-1} = A^{-1} (I_{nT} - \lambda B) A^{-1} \\
= \left( \frac{X' \Omega X}{n} \right)^{-1} (X' Q X) \left( \frac{X' \Omega X}{n} \right)^{-1} \geq 0.
$$
Thus $A^{-1} - \lambda A^{-1} BA^{-1}$ is a positive semidefinite matrix. Therefore, provided $X'TnT \geq 0$,

\[
\begin{align*}
\text{Bias} & \geq 0 \quad \text{if } \frac{\gamma_{1u}}{\gamma_{1\alpha}} \geq \frac{T\sigma_{\alpha}}{\sigma_{u}}, \\
\frac{\partial \text{Bias}}{\partial \gamma_{1u}} & \geq 0, \quad \frac{\partial^2 \text{Bias}}{\partial \gamma_{1\alpha}} \leq 0, \quad \text{and} \quad \frac{\partial^3 \text{Bias}}{\partial \gamma_{1u} \partial \gamma_{1\alpha}} = 0.
\end{align*}
\]

For the nature of decreasing slope of bias with respect to $\gamma_{1\alpha}$, see Tables 4.1 to 4.3. This Bias direction does not hold, that is bias direction is not determined, if each element of $X'TnT$ is not positive or negative.

**Remark 2.2.** Under certain restrictions, there are also some monotonic relations between the Bias and the variances of the error components. Consider the Bias expression in Corollary 2.2, where only $\alpha$ is non-normally distributed. For simplicity, let $k = 1$ and $H = (X'QX)(X'\Omega X/n)^{-1}$. The derivative of the Bias w.r.t. $\sigma_{\alpha}^2$ gives

\[
\frac{\partial \text{Bias}}{\partial \sigma_{\alpha}^2} = -\frac{\gamma_{1\alpha}}{n^2} \left[ H \frac{\partial \lambda (1 - \lambda) \sigma_{\alpha}}{\partial \sigma_{\alpha}^2} + \lambda (1 - \lambda) \sigma_{\alpha} \frac{\partial H}{\partial \sigma_{\alpha}^2} \right] X'T,
\]
where

\[
\partial \lambda (1 - \lambda) \frac{\sigma_a}{\partial \sigma_a^2} = 2\sigma_a^{-1}\lambda (1 - \lambda) (4\lambda - 1) \geq 0 \text{ if } \lambda \geq 1/4
\]
\[
< 0 \text{ if } \lambda < 1/4
\]
\[
\partial H/\partial \sigma_a^2 = 2n^{-1}T\sigma_a^{-2}\lambda^2 (X'QX) (X'\Omega X/n)^{-3} (X'QX) \geq 0.
\]

For \(X'\ell > 0\), if \(\gamma_{1\alpha} < 0\), Bias is an increasing function of \(\sigma_a^2\) when \(\lambda \geq 1/4\). When \(\lambda < 1/4\), the monotonicity is not determined.

Similar result holds for \(\partial Bias/\partial \sigma_a^2\). For \(X'\ell > 0\), if \(\gamma_{1u} < 0\), it is found that Bias is an increasing function of \(\sigma_u^2\) when \(\lambda > 3/4\). When \(\lambda \leq 3/4\), the monotonicity is again not determined.

**Remark 2.3.** Under the non-normality of errors, the MSE depends only on kurtosis. The approximate MSE for normal distribution is greater than or equal to asymptotic MSE, i.e.

\[
\sigma_u^2(X'QX)^{-1} + \frac{2\lambda\sigma_a^2 T}{n^2(T - 1)} \geq \sigma_u^2(X'QX)^{-1}.
\]

The results for approximate MSE result under both normal and non-normal errors in Tables 4.1 to 4.7 suggest that the asymptotic MSE results are generally the same as the approximate MSE results for moderately large samples, at least up to
3 Derivation

Proof of Theorem 2.1. The expansion of the bias vector follows directly from the expansion of $\hat{\beta}_{FGLS}$ around its true value, $\beta$. From (2.7) we know that the expansion of $\hat{\beta}_{FGLS}$ requires the expansion of $\hat{\lambda}$, which further involves the expansion of $\hat{\sigma}_u^2$ and $\hat{\sigma}_\eta^2$. Let us start with the Taylor series expansion of $\hat{\sigma}_u^2$ and $\hat{\sigma}_\eta^2$. From (2.9), we have

$$\hat{\sigma}_u^2 = \frac{u'Qu - u'QX(X'QX)^{-1}X'Qu}{n(T-1) - k}$$

$$= \frac{1}{n(T-1)} \left[ 1 - \frac{k}{n(T-1)} \right]^{-1} \left[ \sigma_u^2 n(T-1) \left( 1 + \frac{v_u}{\sqrt{n}} \right) - \sigma_u^2 v_u^* \right]$$

$$= \frac{1}{n(T-1)} \left[ 1 + \frac{k}{n(T-1)} + \frac{k^2}{n^2(T-1)^2} + \cdots \right]$$

$$\cdot \left[ \sigma_u^2 n(T-1) \left( 1 + \frac{v_u}{\sqrt{n}} \right) - \sigma_u^2 v_u^* \right]$$

$$= \sigma_u^2 \left[ 1 + \frac{v_u}{\sqrt{n}} + \frac{k - v_u^*}{n(T-1)} \right] + O_p(n^{-3/2}), \quad (3.1)$$
where

\[ v_u = \sqrt{n} \left( \frac{u'Qu}{n(T-1)\sigma_u^2} - 1 \right), \quad (3.2) \]

\[ v_u^* = u'QX(X'QX)^{-1}XQu/\sigma_u^2. \quad (3.3) \]

Both \( v_u \) and \( v_u^* \) are \( O_p(1) \). Similarly, we define other \( O_p(1) \) terms frequently used in the proof,

\[ v_{\alpha} = \sqrt{n} \left( \alpha'\alpha\sigma_\alpha^{-2}n^{-1} - 1 \right), \quad (3.4) \]

\[ \epsilon_u = \sqrt{n} \left( u'\tilde{Q}u\sigma_u^{-2}n^{-1} - 1 \right), \quad (3.5) \]

\[ v_{\alpha}^* = \alpha'D'X(X'QX)^{-1}X'D\alpha/\sigma_\eta^2, \quad (3.6) \]

\[ \epsilon_u^* = u'\tilde{Q}X(X'\tilde{Q}X)^{-1}X'\tilde{Q}u/\sigma_\eta^2, \quad (3.7) \]

\[ v_{\alpha u} = \frac{u'D\alpha}{\sqrt{n}\sigma_\eta^2}, \quad (3.8) \]

\[ v_{\alpha u}^* = \alpha'D'X(X'\tilde{Q}X)^{-1}X'\tilde{Q}u/\sigma_\eta^2, \quad (3.9) \]

For \( \sigma_\eta^2 \), we have

\[ u'\tilde{Q}w = \alpha'D'\tilde{Q}D\alpha + u'\tilde{Q}u + 2u'\tilde{Q}D\alpha \]

\[ = nT\sigma_\alpha^2 \left( 1 + v_\alpha/\sqrt{n} \right) + n\sigma_\alpha^2 \left( 1 + \epsilon_\alpha/\sqrt{n} \right) + 2\sqrt{n}\sigma_\eta^2 v_{\alpha u} \]
\[ w' \bar{Q}X (X' \bar{Q}X)^{-1} X' \bar{Q} w = \sigma_\eta^2 (v_\alpha^* + \epsilon_u^* + 2v_{\alpha u}^*) \].

(3.11)

\[ \hat{\sigma}_\eta^2 = \sigma_\eta^2 \left[ 1 + \frac{(1 - \lambda) v_\alpha + \lambda \epsilon_u + 2v_{\alpha u}}{\sqrt{n}} + \frac{k - (v_\alpha^* + \epsilon_u^* + 2v_{\alpha u}^*)}{n} \right] + O_p(n^{-3/2}). \]

(3.12)

Using (3.1) and (3.12), it can be verified that

\[ \hat{\lambda} = \lambda \left[ 1 + \frac{f}{\sqrt{n}} + \frac{f^* - f v_u + f^2}{n} \right] + O_p(n^{-3/2}), \]

(3.13)

where

\[ f = v_u - (1 - \lambda) v_\alpha - \lambda \epsilon_u - 2v_{\alpha u} \]

(3.14)

\[ f^* = v_\alpha^* + \epsilon_u^* + 2v_{\alpha u}^* - v_u^*/(T - 1) - k(T - 2)/T \]

(3.15)
Multiplying both sides of (3.13) by $\sqrt{n}$ and rearranging the equation gives

$$\sqrt{n}(\hat{\lambda} - \lambda) = \lambda f + \lambda \left( f^* - f v_u + f^2 \right) / \sqrt{n} + O_p(n^{-1}). \tag{3.16}$$

Now define

$$\delta = \sqrt{n}(\hat{\lambda} - \lambda) \tag{3.17}$$

so that $\delta^2 = \lambda^2 f^2 + O_p(n^{-1/2})$. Using the above definition, we have

$$\hat{\Omega} = \Omega + \bar{Q}\delta / \sqrt{n}, \tag{3.18}$$

$$X'\hat{\Omega}X/n = A + B\delta / \sqrt{n}. \tag{3.19}$$

Now plug (3.18) and (3.19) into (2.7) and multiply both sides by $\sqrt{n}$ we have

$$\sqrt{n}(\hat{\beta}_{GLS} - \beta) = \left( A + B\delta / \sqrt{n} \right)^{-1} \left[ X'(\Omega + \bar{Q}\delta / \sqrt{n})w / \sqrt{n} \right] \tag{3.20}$$

$$= A^{-1} \left( X'\Omega w / \sqrt{n} \right) + A^{-1} P_1 w\delta / \sqrt{n} + A^{-1} P_2 w\delta^2 / n,$$

where $P_1$ is as given in Theorem 2.1 and $P_2 = -BA^{-1}P_1$. It can be easily verified that $Q_{t_{nT}} = t_{nT}$, $P_1X = P_2X = 0$, $P_1t_{nT} = (X' - \lambda BA^{-1}X')t_{nT}/\sqrt{n}$, $P_1\bar{Q}X/\sqrt{n} = B - \lambda BA^{-1}B$, and $P_1QX/\sqrt{n} = -P_1QX/\sqrt{n}$. 

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Then using (3.17) we get

\[
\sqrt{n}(\hat{\beta}_{FGLS} - \beta) = \xi_0 + \xi_{-1/2} + \xi_{-1} + O_p(n^{-3/2}),
\]

(3.21)

where

\[
\xi_0 = A^{-1} \left( X' \Omega w / \sqrt{n} \right), \quad \xi_{-1/2} = \lambda A^{-1} P_1 w_f / \sqrt{n},
\]

\[
\xi_{-1} = \lambda^2 A^{-1} P_2 w_f^2 / n + \lambda A^{-1} P_1 w (f^* - f_v + f^2) / n.
\]

Taking expectation of (3.21) to obtain the bias vector up to \(O_p(n^{-1/2})\)

\[
E \left[ \sqrt{n}(\hat{\beta}_{FGLS} - \beta) \right] = E\xi_0 + E\xi_{-1/2}
\]

\[
= \lambda A^{-1} P_1 E(w_f) / \sqrt{n}.
\]

(3.22)

It is easy to verify that \(E\xi_0 = 0\). Now let us evaluate \(E(w_f) = E(D\alpha + u)f = DE(\alpha f) + E(uf)\). From (3.14) we get

\[
DE(\alpha f) = DE \left[ \sigma_u^{-2} u' \left( Q / (T - 1) - \lambda \hat{Q} \right) u\alpha / \sqrt{n} - (1 - \lambda) \sigma_\alpha^{-2} \alpha' \alpha \alpha / \sqrt{n}
\]

\[
- 2\sigma_\eta^{-2} u' D \alpha \alpha / \sqrt{n} \right]
\]

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\[
\begin{align*}
\mathbb{E}(u f) &= E[u (v_u - \lambda \epsilon_u)] \\
&= \sigma_u^2 \left[ E(u' u) / (T - 1) - \frac{\lambda (T - 1)}{T - 1} E(u' \tilde{Q} u) \right] / \sqrt{n} \\
&= \gamma_{1u} \sigma_u (1 - \lambda) T^{-1} n^{-1/2} t_{nT}. \quad (3.24)
\end{align*}
\]

Combine (3.23) and (3.24) we have

\[
\mathbb{E}(w f) = (1 - \lambda) (\gamma_{1u} \sigma_u / T - \gamma_{1a} \sigma_a) t_{nT} / \sqrt{n}. \quad (3.25)
\]

Hence substituting (3.25) in (3.22) we get the bias result in *Theorem 2.1*.

The mean square error matrix up to order \(O(n^{-1})\) is

\[
\begin{align*}
\mathbb{E} \left[ n(\hat{\beta}_{FGS} - \beta)(\hat{\beta}_{FGS} - \beta)' \right] &= E (\xi_0 \xi_0') + E (\xi_0 \xi_{-1/2}') + E (\xi_{-1/2}' \xi_0) \\
&\quad + E (\xi_{-1/2}' \xi_{-1/2}) \\
&\quad + E (\xi_0 \xi_{-1} + \xi_{-1} \xi_0') \quad \text{.} \quad (3.26)
\end{align*}
\]
where from (3.21) we have

\[
E(\xi_0') = n \sigma_u^2 (X'QX)^{-1},
\]
\[
E(\xi_0' - \frac{1}{2}) = \lambda A^{-1} X'\Omega E(wu' f) P_1' A^{-1} / n,
\]
\[
E(\xi_{-1}' - \frac{1}{2}) = \lambda^2 A^{-1} P_1 E(wu' f^2) P_1' A^{-1} / n,
\]
\[
E(\xi_{-1}' - 1) = \lambda^2 A^{-1} (X'\Omega/\sqrt{n}) E(wu' f^2) P_2' A^{-1} / n
\]
\[+ \lambda A^{-1} (X'\Omega/\sqrt{n}) E((f^* - f v_e + f^2) uu') P_1' A^{-1} / n.\]

Consider the expectation

\[
E(wu' f) = DE(f\alpha') D' + DE(f\alpha') D' + E(fu' D') + E(\beta uu'), \quad (3.27)
\]

where

\[
E(f\alpha') = E(u' (Q/(T-1) - \lambda Q) u \sigma_u^{-2} / \sqrt{n}) E(\alpha' \alpha')
\]
\[= (1 - \lambda) \alpha^2 E(\alpha' \alpha' / \sqrt{n})
\]
\[= (1 - \lambda) (2 + \gamma_2 \sigma_\alpha^2 I_n / \sqrt{n},
\]
\[
E(fu') = -2 \sigma_\eta^{-2} E(\alpha' \alpha') D'E(uu') / \sqrt{n}
\]
\[= -2 \lambda \sigma_\alpha^2 \sigma_\eta^{-2} D' / \sqrt{n}.
\]
\[ E (f u \alpha') = -2 \lambda \sigma_{\alpha}^2 \sigma_{\eta}^{-2} D / \sqrt{n}, \]
\[ E(f u u') = \sigma_{u}^{-2} E [u' (Q / (T - 1) - \lambda \bar{Q}) u \cdot uu'] / \sqrt{n} \]
\[ - (1 - \lambda) \sigma_{\alpha}^{-2} E (\alpha \alpha') E (uu') / \sqrt{n} \]
\[ = \sigma_{u}^2 [(1 - \lambda) \gamma_{2u} I_{nT} / T \]
\[ + 2 (T - 1) Q - 2 \lambda \bar{Q}] / \sqrt{n}. \]

Now substitute these four terms into \( E (wu'f) \), and we get

\[ E (wu'f) = \sigma_{u}^2 [(1 - \lambda) \gamma_{2u} I_{nT} / T - (1 - \lambda) \gamma_{2u} \bar{Q} / \lambda] + 2Q / (T - 1) \]
\[ - 2 \bar{Q} / \lambda] / \sqrt{n}. \]  

Next let us define \( Z_u = u / \sigma_u, Z_\alpha = \alpha / \sigma_\alpha \), and the first four moments of the elements of \( Z_u \) and \( Z_\alpha \) are given in the Appendix. Then

\[ E (wu'f^2) = DE (f^2 \alpha \alpha') D' + DE (f^2 \alpha u') + E (f^2 u \alpha') D' + E (f^2 uu'). \]

(3.29)
Consider the first term on the right-hand side of (3.29) we note that

\[
E (\alpha' f'^2) = E \left[ \alpha \alpha' \left( v_u^2 + (1 - \lambda)^2 v_\alpha^2 + \lambda^2 \varepsilon_u^2 + 4v_{\alpha u}^2 - 2(1 - \lambda) v_u \varepsilon_u \right) \right]
\]
\[
+ E \left[ \alpha \alpha' \left( -2\lambda v_u \varepsilon_u - 4v_u v_{\alpha u} + 2\lambda (1 - \lambda) \varepsilon_u \right) \right]
\]
\[
+ E \left[ \alpha \alpha' \left( 4(1 - \lambda) v_u v_{\alpha u} + 4\lambda \varepsilon_u v_{\alpha u} \right) \right]
\]
\[
= \sigma_u^2 I_n E \left( v_u^2 + \lambda^2 \varepsilon_u^2 - 2\lambda v_u \varepsilon_u \right) + E \left[ \alpha \alpha' \left( -4v_u v_{\alpha u} + 4\lambda \varepsilon_u v_{\alpha u} \right) \right]
\]
\[
+ E \left[ \alpha \alpha' \left( 4v_{\alpha u}^2 - 2(1 - \lambda) v_u v_{\alpha} + 2\lambda (1 - \lambda) \varepsilon_u \right) \right]
\]
\[
+ E \left[ \alpha \alpha' \left( 1 - \lambda \right)^2 v_\alpha^2 \right] + 4(1 - \lambda) E \left[ \alpha \alpha' v_u v_{\alpha u} \right]
\]
\[
= I + II + III + IV + V, \quad (3.30)
\]

where

\[
I = \sigma_\alpha^2 I_n E (v_u - \lambda \varepsilon_u)^2
\]
\[
= n \sigma_\alpha^2 I_n E \left[ Z_u' \left( Q / (T - 1) - \lambda \bar{Q} \right) Z_u / n - (1 - \lambda) \right]^2
\]
\[
= \sigma_\alpha^2 I_n \left[ \gamma_{2u} (1 - \lambda)^2 / T + 2 / (T - 1) + 2\lambda^2 \right]
\]
\[ II = -4E \left[ \alpha \alpha' v_{\alpha u} (v_u - \lambda \epsilon_u) \right] \]
\[ = -4 \frac{\sigma_u}{\sigma_{\eta}} E_{\alpha} \left[ \alpha \alpha' \cdot \alpha' D' E Z_{\alpha} (Z_u \cdot Z_u' (Q / (T - 1) - \lambda \bar{Q}) Z_u / n) \right] \]
\[ = -4 (1 - \lambda) \gamma_{1u} \gamma_{1\alpha} \sigma_{\alpha} \sigma_{\eta}^2 \sigma_{\eta}^{-2} I_n / n = O \left( n^{-1} \right) , \]
\[ III = E \left[ \alpha \alpha' \left( 4v_{\alpha u}^2 - 2 (1 - \lambda) v_{\alpha} (v_u - \lambda \epsilon_u) \right) \right] \]
\[ = E \left[ \alpha \alpha' \left( 4 (u'D \alpha \sigma_{\eta}^2 / \sqrt{n})^2 - 2 (1 - \lambda) n \left( \alpha \alpha' \sigma_{\alpha}^2 / n - 1 \right) \right. \right. \]
\[ \cdot \left. \left. u' (Q/T - 1 (T - 1) - \lambda \bar{Q}) u \sigma_{\eta}^{-2} / n - (1 - \lambda) \right) \right] \]
\[ = 4 \lambda (1 - \lambda) \sigma_{\alpha}^2 I_n + O \left( n^{-1} \right) , \]
\[ IV = E \left[ \alpha \alpha' (1 - \lambda)^2 n \left( \alpha' \alpha \sigma_{\alpha}^2 / n - 1 \right)^2 \right] \]
\[ = (1 - \lambda)^2 \sigma_{\alpha}^2 E \left[ Z_{\alpha} Z_{\alpha}' \left( (Z_{\alpha}' Z_{\alpha})^2 / n - 2 Z_{\alpha} Z_{\alpha}' + n \right) \right] \]
\[ = (1 - \lambda)^2 \sigma_{\alpha}^2 (\gamma_{2\alpha} + 2) I_n , \]
\[ V = 0. \]

Substitute the above five results in (3.30), we have

\[ DE \left( \alpha \alpha' f^2 \right) D' = T \sigma_{\alpha}^2 \left[ (1 - \lambda)^2 (\gamma_{2u}/T + \gamma_{2\lambda}) + 2T / (T - 1) \right] \bar{Q}. \] (3.31)
In the second term on the right hand side of (3.29)

\[ E(\alpha u' f^2) = E[\alpha u' (v_u - \lambda \epsilon_u)^2] - 4E[\alpha u' v_{\alpha u} (v_u - \lambda \epsilon_u)] + E[\alpha u' (4v_{\alpha u}^2 - 2(1 - \lambda) v_{\alpha} (v_u - \lambda \epsilon_u))] + E[\alpha u' (1 - \lambda)^2 v_{\alpha}^2] + 4(1 - \lambda) E(\alpha u' v_{\alpha} v_{\alpha u}), \quad (3.32) \]

where

\[ I = 0, \]

\[ II = -4E[\alpha u'u'D\alpha (u' (Q/(T - 1) - \lambda \bar{Q}) u/s^2/n - (1 - \lambda))]/s^2 \]

\[ = -4\sigma_2^2\sigma_1^2 D'E[Z_u Z'_u (Z'_u (Q/(T - 1) - \lambda \bar{Q}) Z_u/n - (1 - \lambda))] /s^2 \]

\[ = -4\sigma_2^2\sigma_1^2 D' \left[ \gamma_{2u} (1 - \lambda) I_n T n^{-1} T^{-1} + 2 (Q/(T - 1) - \lambda \bar{Q}) /n \right] /s^2 \]

\[ = O(n^{-1}), \]

\[ III = E[\alpha u' (4u'D\alpha' D'u/s^2/n - 2(1 - \lambda)n (\alpha \alpha' \sigma_1^2/n - 1) \cdot (u' (Q/(T - 1) - \lambda \bar{Q}) u/s^2/n - (1 - \lambda)))] \]

\[ = O(n^{-1}), \]

\[ IV = 0, \]
\[ V = 4 (1 - \lambda) E \left[ \alpha u' \left( \alpha \sigma_\alpha^2 / n - 1 \right) u' D \sigma_\eta^2 \right] \]
\[ = 4 (1 - \lambda) \sigma_u^2 \sigma_\eta^2 E \left[ \alpha u' \left( \alpha \sigma_\alpha^2 / n - 1 \right) \right] D' \]
\[ = 4 (1 - \lambda) \sigma_u^2 \sigma_\alpha^2 \sigma_\eta^2 \left( \gamma_2 \sigma_\lambda \sigma_\theta \sigma_\eta \sigma_\eta \sigma_\eta \right) \frac{1}{n} \frac{1}{n - 1} \frac{1}{n - 2/2} \frac{1}{n - \lambda} \frac{D'}{D} = O(n^{-1}). \]

Substitute above five results into (3.32) and we get
\[
E (\alpha u' f^2) = O(n^{-1}). \tag{3.33}
\]

The fourth term on the right-hand side of (3.29) is
\[
E (f^2 u u') = E \left[ u u' (v_u - \lambda \epsilon_u)^2 \right] - 4E \left[ u u' (v_{uu} (v_u - \lambda \epsilon_u)) \right] \]
\[ + E \left[ u u' (4 v_{uu}^2 - 2 (1 - \lambda) v_{uu} (v_u - \lambda \epsilon_u)) \right] \]
\[ + E \left[ u u' \left( (1 - \lambda) v_{uu} \right) \right] + 4E \left[ u u' \left( (1 - \lambda) v_{uu} \right) \right] \]
\[ = I + II + III + IV + V, \tag{3.34}
\]

where
\[
I = E \left[ u u' n \left( u' (Q / (T - 1) - \lambda \bar{Q}) u \sigma_u^2 / n - (1 - \lambda) \right)^2 \right] \]
\[ = \sigma_u^2 E \left[ n^{-1} Z_u Z_{u'} \left( (Z_u' (Q / (T - 1) - \lambda \bar{Q}) Z_u)^2 \right) \right]
\]

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\[-2n (1 - \lambda) Z_u' (Q / (T - 1) - \lambda \bar{Q}) Z_u + n^2 (1 - \lambda)^2 \]
\[= \sigma_u^2 [(1 - \lambda)^2 \gamma_{2u} / T + 2 (1 / (T - 1) + \lambda^2)] I_{nT},\]

\[II = 0,\]

\[III = 4\sigma_u^2 \sigma_\eta^{-4} E (uu' \cdot u'DD'u) / n\]
\[= 4\sigma_u^2 \sigma_\eta^{-4} T I_{nT}\]
\[= 4(1 - \lambda) \lambda \sigma_u^2 I_{nT} + O (n^{-1}),\]

\[IV = (1 - \lambda)^2 E \left[uu'n (\alpha\alpha' \sigma_\alpha^{-2} / n - 1)^2\right]\]
\[= \sigma_u^2 (1 - \lambda)^2 nE (\alpha\alpha' \sigma_\alpha^{-2} / n - 1)^2 I_{nT}\]
\[= \sigma_u^2 (1 - \lambda)^2 (\gamma_{2\alpha} + 2) I_{nT},\]

\[V = 4(1 - \lambda) E \left[uu'u'D\alpha\sigma_\eta^{-2} (\alpha\alpha' \sigma_\alpha^{-2} / n - 1)\right]\]
\[= 4(1 - \lambda) \sigma_\alpha^{-2} \sigma_\eta^{-2} E (uu' \cdot u'D\alpha \cdot \alpha') / n = O(n^{-1}).\]

Hence

\[E (uu' f^2) = \sigma_u^2 [(1 - \lambda)^2 (\gamma_{2u} / T + \gamma_{2\alpha}) + 2T / (T - 1)] I_{nT},\]  \hspace{1cm} (3.35)
and (3.29) can be written as

\[ E(ww'f^2) = \sigma_u^2 \left[ (1 - \lambda)^2 (\gamma_{2u}/T + \gamma_{2\alpha}) + 2T/(T-1) \right] \left( \frac{1-\lambda}{\lambda} \bar{Q} + I_{nt} \right). \]

(3.36)

Next, let us consider \( E(f^*ww') \) in (3.26)

\[
E(f^*ww') = E(f^*uu') + DE(f^*\alpha u') + E(f^*u\alpha')D' + DE(f^*\alpha\alpha')D' \\
= I + II + III + IV,
\]

(3.37)

where

\[
I = E[(\epsilon^*_u + \epsilon^*_u + 2\epsilon^*_uu - \epsilon^*_u/(T-1) - k(T-2)/(T-1)) uu'] \\
= \sigma_u^2 \left[ \sigma^2_{\alpha} \sigma^{-2}_{\eta} kT - k(T-2)/(T-1) \right] + E[(\epsilon^*_u - \epsilon^*_u/(T-1)) uu'] \\
= \sigma_u^2 \gamma_{2u} \left[ (\lambda (I_{nt} \odot \bar{Q}X(X'\bar{Q}X)^{-1}X') + (I_{nt} \odot QX(X'QX)^{-1}X'/Q) / (T - 1) \right] + \sigma_u^2 [2(\lambda QX(X'QX)^{-1}X'Q - QX(X'QX)^{-1}X'/Q) / (T - 1)],
\]

\[
II = DE[2\epsilon^*_uu \cdot \alpha u'] \\
= 2DE \left[ \epsilon\alpha' D'X(X'\bar{Q}X)^{-1}X'\bar{Q}uu'\sigma^{-2}_{\eta} \right] \\
= 2(1-\lambda) \sigma^2_u \bar{Q}X(X'\bar{Q}X)^{-1}X'\bar{Q} = III,
\]
IV = DE \left[ (v_{\alpha}^* + \epsilon_u^* + 2v_{\alpha u}^* - v_u^* (T - 1) - k (T - 2) / (T - 1) ) \alpha \alpha' \right] D' \\
= DE \left[ v_{\alpha}^* \alpha \alpha' \right] D' + \sigma_{\alpha}^2 DD' (k \lambda - k / (T - 1) - k (T - 2) / (T - 1)) \\
= (1 - \lambda)^2 \lambda^{-1} \gamma_{2a} D \left( I_n \odot D'X(X'\bar{Q}X)^{-1}X'D \right) D' / T^2 \\
\quad + 2 (1 - \lambda)^2 \lambda^{-1} \sigma_{\alpha}^2 \bar{Q}X(X'\bar{Q}X)^{-1}X'\bar{Q}.

Thus

\[
E (f^*u w') = \sigma_{\alpha}^2 \left[ \gamma_{2a} (\lambda \left( I_n \odot \bar{Q}X(X'\bar{Q}X)^{-1}X'\bar{Q} \right) \\
\quad - (I_n \odot \bar{Q}X(X'\bar{Q}X)^{-1}X'Q) / (T - 1) \right] \\
\quad + \sigma_{\alpha}^2 \left[ (1 - \lambda)^2 \lambda^{-1} \gamma_{2a} D \left( I_n \odot D'X(X'\bar{Q}X)^{-1}X'D \right) D' \\
\quad - 2QX(X'\bar{Q}X)^{-1}X'Q / (T - 1) \right] \\
\quad + \sigma_{\alpha}^2 \left[ 2\lambda^{-1} \bar{Q}X(X'\bar{Q}X)^{-1}X'\bar{Q} \right]. \tag{3.38}
\]

Consider \( E (f_u w w') \) in (3.26)

\[
E (f_u w w') = E (f_u u w') + E (f_u u \alpha' ) D' + DE (f_u \alpha u') \\
\quad + DE (f_u \alpha \alpha') D' = I + II + III + IV \tag{3.39}
\]

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where

\begin{align*}
I &= E[(v_u - \lambda \epsilon_u) v_u \cdot uu'] \\
&= n \sigma_u^2 E[(Z_u'(Q/(n(T - 1)) - \lambda \bar{Q}/n) Z_u - (1 - \lambda)) \\
&\quad \cdot (Z'_u Q Z_u n^{-1}/(T - 1) - 1) Z'_u Z_u] \\
&= \sigma_u^2 [2/(T - 1) + \gamma_{2u} (1 - \lambda)/T] I_n T, \\
II &= E[(v_u - (1 - \lambda) v_\alpha - \lambda \epsilon_u - 2v_{\alpha u}) v_u \cdot u \alpha'] D' \\
&= E[-(1 - \lambda) v_\alpha v_u \cdot u \alpha'] D' + E(-2v_{\alpha u} v_u \cdot u \alpha') D' \\
&= -(1 - \lambda) E(v_u u) E(v_\alpha \alpha') D' - 2\sigma_\alpha^2 \sigma_\eta^{-2} E(v_u uu') DD'/\sqrt{n} \\
&= O(n^{-1}) = III, \\
IV &= DE[(v_u - (1 - \lambda) v_\alpha - \lambda \epsilon_u - 2v_{\alpha u}) v_u \alpha \alpha'] D' \\
&= \sigma_\alpha^2 DE[(v_u - \lambda \epsilon_u) v_u] D' - 2DE(v_{\alpha u} v_u \alpha \alpha') D' \\
&= \sigma_u^2 (1 - \lambda) \lambda^{-1}[\gamma_{2u} (1 - \lambda) T^{-1} + 2/(T - 1)] \bar{Q} + O(n^{-1}).
\end{align*}

Therefore

\begin{equation}
E(fv_u w w') = \sigma_u^2 \lambda^{-1}[\gamma_{2u} (1 - \lambda)/(T + 2/(T - 1)) (\bar{Q} + \lambda Q). \tag{3.40}
\end{equation}
Plugging (3.28), (3.36), (3.38), and (3.40) into (3.26) we have

\[
E \left( \xi_0 \xi'_{-1/2} \right) = \frac{\lambda}{n} A^{-1} X' \Omega E (ww'f) P_1' A^{-1}
\]
\[
= -\lambda \sigma_u^2 [\gamma_{2u} (1 - \lambda)^2 T^{-1} + \gamma_{2u} (1 - \lambda)^2 T^{-1} + 2T/(T-1)] \Delta / n
\]
\[
= E \left( \xi_{-1/2} \xi'_0 \right),
\]
\[
E(\xi_{-1/2} \xi'_{-1/2}) = \lambda^2 A^{-1} P_1 E (ww'f^2) P_1' A^{-1} / n
\]
\[
= \lambda^2 \sigma_u^2 \left[ (1 - \lambda)^2 (\gamma_{2u} T^{-1} + \gamma_{2u}) + 2T/(T-1) \right] \Delta / n,
\]
\[
E \left( \xi_0 \xi'_{-1} \right) = C + \frac{2\lambda \sigma_u^2 T}{n (T - 1)} \Delta,
\]
\[
E \left( \xi_{-1} \xi'_0 \right) = C' + \frac{2\lambda \sigma_u^2 T}{n (T - 1)} \Delta.
\]

Using these in (3.26) the MSE result in Theorem 2.1 follows. Q.E.D.

### 4 Numerical Results

In this section we provide a numerical study of the behavior of analytical Bias and MSE under non-normality. The data generating process is specified as follows

\[
y_{it} = x_{it} \tilde{\beta} + \alpha_i + u_{it}.
\]
$x_{it}$ are generated via the method of Nerlove (1971)

\begin{align*}
x_{it} &= 0.1t + 0.5x_{it-1} + w_{it}, \\
x_{i0} &= 10 + 5w_{i0}, \\
w_{it} &\sim U \left[ -\frac{1}{2}, \frac{1}{2} \right].
\end{align*}

We omit the constant term and consider the data generating process described in Corollary 2.2 and Corollary 2.3. For Corollary 2.2, we let $\beta = 0.5$. $u_{it} \sim \mathcal{IIN}(0, 0.36)$, which implies $\gamma_{1u} = \gamma_{2u} = 0$. $\alpha_i$ are generated by Johnson’s (1949) $S_u$ system, introducing non-normality to our data generating process. The non-normal $\alpha_i$ is generated by transforming a standard normal random variable $\varepsilon_i$

$$\alpha_i^* = \sinh \left( \frac{\varepsilon_i - \theta_1}{\theta_2} \right),$$

and letting $\alpha_i$ be the standardized version of $\alpha_i^*$ with zero mean and variance is one.

Different values of $(\theta_1, \theta_2)$ gives different values of the skewness and kurtosis of the random variable $\alpha_i^*$. Define $\omega = \exp(\theta_2^2)$ and $\psi = \theta_1/\theta_2$ and the four
moments of $\alpha_i$ are given by

\begin{align*}
E(\alpha_i^*) &= \mu_\alpha = -\omega^{1/2} \sinh(\psi), \\
E(\alpha_i^* - \mu_\alpha)^2 &= \frac{1}{2} (\omega - 1) [\omega \cosh(2\psi) + 1], \\
E(\alpha_i^* - \mu_\alpha)^3 &= -\frac{1}{4} \omega^{1/2} (\omega - 1)^2 [\omega (\omega + 2) \sinh(3\psi) + 3 \sinh(\psi)], \\
E(\alpha_i^* - \mu_\alpha)^4 &= \frac{1}{8} (\omega - 1)^2 [\omega^2 (\omega^4 + 2\omega^3 + 3\omega^2 - 3) \cosh(4\psi) \\
&\quad + 4\omega^2 (\omega + 2) \cosh(2\psi) + 3 (2\omega + 1)].
\end{align*}

From this we get skewness $\gamma_{1\alpha} = E(\alpha_i^* - \mu_\alpha)^3 / (E(\alpha_i^* - \mu_\alpha)^2)^{3/2}$ and excess kurtosis $\gamma_{2\alpha} = E(\alpha_i^* - \mu_\alpha)^4 / (E(\alpha_i^* - \mu_\alpha)^2)^2 - 3$. In Tables 4.1 to 4.3, $\theta_1$ is set to be 4 and $\theta_2 \in [1.5, 3]$. This combination of $\theta_1$ and $\theta_2$ gives a moderate interval for the variance of $\alpha_i^*$, from 0.5 to 45. For Corollary 2.3, we apply the same method to the generation of non-normal $u_{it}$, and let $\alpha_i \sim IIN(0, 4)$. In order to investigate the finite sample behavior of Bias and MSE, we let $n = 10$ and $T = 5$. We replicate the experiment 1000 times for each pair of $(\theta_1, \theta_2)$.

When only $\alpha$ is non-normal, we note that from Table 4.1 that the MSE changes with $\gamma_{2\alpha}$. Generally, for some large $\gamma_{2\alpha}$, approximate MSE is less than asymptotic MSE while for some small $\gamma_{2\alpha}$, approximate MSE is greater than asymptotic MSE. Thus the use of the asymptotic MSE, when the sample is small or mod-
erately large, will provide an under estimation or over estimation depending on the magnitude of $\gamma_{2u}$. Further the t-ratios for hypothesis testing, based on asymptotic MSE, may provide under or over rejection of the null hypothesis. When the sample is moderately large (Table 4.2) we get similar results, but the asymptotic MSE is the same as the approximate MSE up to 4 digits. However, for the cases when only $u_{it}$ is non-normal we see from Table 4.4 that the approximate MSE is greater than the asymptotic MSE for all values of $\gamma_{2u}$. Thus, in this case, the use of asymptotic MSE in practice, will generally provide underestimation of MSE and t-ratios may falsely reject the null hypothesis. For moderately large samples in Table 4.5, the approximate MSE is still greater than the asymptotic MSE, but they are the same up to 4 digits. Thus, when either alpha or u is non-normally distributed, we observe that while the use of the asymptotic MSE may provide under or over estimation of the MSE, the asymptotic MSE estimates the approximate MSE accurately since they are the same up to three or four digits, especially for moderately large samples.

In Remark 2.1, Bias is found to be a decreasing function of $\gamma_{1a}$ and an increasing function of $\gamma_{1u}$, which is consistent with the results seen in Tables 4.1-4.6. The monotonic relations between Bias and variances of the error components in Remark 2.2 are shown numerically in Tables 4.8-4.11, where in Tables 4.8-4.9 we
fix $\sigma_u$ and increase $\sigma_\alpha$ and in Tables 4.10-4.11 we fix $\sigma_\alpha$ and increase $\sigma_u$.

We also simulate the different $ns$ for the same $T$ and vice versa. The results presented here are for $T = 5$ with $n = 10, 50$ and for $n = 10$ with $T = 5, 50$. The results for other values of $n$ and $T$ are available from the authors upon request, and they give the similar conclusions. When $\alpha$ is non-normal, the maximum relative bias, $E (\hat{\beta} - \beta) / \beta$, decreases from 2.7% to 0.5% when $n$ changes from 10 to 50 with $T = 5$; and it decreases from 2.7% to 0.3% when $n = 10$ and $T$ changes from 5 to 50. When $u$ is non-normal, the maximum relative bias changes from 0.4% to 0.08% for the change of $n$ from 10 to 50 with $T = 5$; and from 0.4% to 0.004% for $n = 10$ when $T$ changing from 5 to 50. Thus the order of bias is not very significant, further, it is found that for a fixed $T$, e.g. $T = 5$, when $n$ is large enough, for example,50, the approximate bias is practically zero. These results are consistent with the results in Maddala and Mount (1973). For the MSE, when $\alpha$ is non-normal and $T$ is fixed at 5, the approximate MSE is equal to asymptotic MSE up to the third digit when $n = 10$, but up to the fourth digit when $n = 50$. For the case when $n$ is fixed at 10 and $T$ changes from 5 to 50, the two MSEs are the same up to three digits. Similar results hold for the case when $u$ is non-normally distributed.

Next we consider the DGPs with both error components are non-normally
distributed and have large variances in small sample. It is found in such cases the
relative bias can be large and asymptotic MSE may not be very accurate. Tables
4.12-4.14 give some examples. Most tables show that the approximate bias is not
negligible. The range of relative bias in Table 4.12 is [3%, 8.7%] and it increases
to [10%, 28%] in Table 4.14. The approximate and asymptotic MSEs can be
different even at the first digit, as shown in the first row of Table 4.14.

In Table 4.7, both $\alpha$ and $u$ are normal, where $u_{it} \sim IIN(0, 0.36)$ and $\alpha_i$ has
zero mean and changing variance. $\gamma_{1\alpha} = \gamma_{2\alpha} = \gamma_{1u} = \gamma_{2u} = 0$. In this case, the
approximate MSE is always larger than asymptotic MSE, and this is consistent
with the results in Corollaries 2.1 and Remark 2.3. However, the difference in the
approximate and asymptotic MSEs is the same up to 5 digits.

5 Conclusion

In this paper, we study the finite sample properties of the FGLS estimators in
random-effects model with non-normal errors. We derive the asymptotic expan-
sion of the Bias and MSE up to $O(n^{-1})$ and $O(n^{-2})$, respectively.

Firstly, the Bias depends only on skewness coefficient. Bias is zero for sym-
metric distributions or for distributions satisfying $\gamma_{1u}/\gamma_{1\alpha} = T\sigma_{\alpha}/\sigma_u$. We find
Bias is a nondecreasing function of $\gamma_{1u}$ and a non-increasing function of $\gamma_{1o}$ provided $X'\nu_{nT} \geq 0$. Under certain parameter restrictions, Bias is also found to be monotonic functions of variances of the error components.

Secondly, the MSE depends only on the kurtosis coefficient. The approximate MSE can be greater or smaller than asymptotic MSE. The statistical inference based on using the asymptotic MSE can be quite accurate when variances of the error components are small since it is the same as the approximate MSE, under the normality as well as a non-normal distribution considered, up to three or four digits, especially for moderately large samples. However, when those variances are large, asymptotic results can give inaccurate results.
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Appendix

The following results have been repeatedly used in the derivation in Section 3:

Let $G_1$ and $G_2$ be two $nT \times nT$ idempotent matrices with nonstochastic elements such that

\[
\begin{align*}
tr(G_1) &= ng_1, \\
tr(G_2) &= ng_2, \\
tr(G_{12}) &= ng_{12}.
\end{align*}
\]

Assuming $G_1$ and $G_2$ to be symmetric matrices and $Z$ to be an $nT \times 1$ random vector whose elements are i.i.d. with the first four moments given as\(^1\)

\[
Ez_j = 0, Ez_j^2 = 1, Ez_j^3 = \gamma_{1z}, Ez_j^4 = \gamma_{2z} + 3, \quad j = 1, \ldots, nT
\]

Then we have

\(^1\)If $Z = Z_\alpha$, the dimension changes to $n$, which implies $T = 1$ in the following results.
Further, if the diagonal elements of $G_1$ are equal and those of $G_2$ are also equal, we have

\begin{align}
E(Z'G_1Z) &= \gamma_{1z}(I_{nT} \odot G_1)_{nT} \quad \text{(A.1)} \\
E(Z'G_1Z'Z) &= \gamma_{2z}(I_{nT} \odot G_1) + tr(G_1) + 2G_1 \quad \text{(A.2)}
\end{align}

Notice that results (A.1) to (A.4) are exact while the result (A.5) is given up to order $O(n^{-1})$ only as it suffices for the present purpose.
References


Nerlove, M., 1967. Experimental evidence on the estimation of dynamic eco-
nomic relations from a time series of cross-sections. Economic Studies Quarterly 18, 42–74.


Table 4.1

\[
\begin{array}{cccccc}
\theta_2 & \gamma_{1\alpha} & \gamma_{2\alpha} & \text{Approx Bias} & \text{Asym Bias} & \text{Approx MSE} & \text{Asym MSE} \\
1.5 & -2.65 & 14.65 & 0.013656 & 0 & 0.002601 & 0.002902 \\
2.0 & -1.70 & 5.64 & 0.008769 & 0 & 0.002848 & 0.002902 \\
2.5 & -1.23 & 2.93 & 0.006350 & 0 & 0.002921 & 0.002902 \\
3.0 & -0.94 & 1.74 & 0.004853 & 0 & 0.002954 & 0.002902 \\
\end{array}
\]

\( \alpha \) is non-normal and \( u \) is normal, \( \sigma_\alpha=1, \sigma_u=0.6 \)
Table 4.2

\( n = 50, \ T = 5 \) \( \alpha \) is non-normal and \( u \) is normal \( \sigma_\alpha = 1, \sigma_u = 0.6 \)

<table>
<thead>
<tr>
<th>( \theta_2 )</th>
<th>( \gamma_{1\alpha} )</th>
<th>( \gamma_{2\alpha} )</th>
<th>Approx Bias</th>
<th>Asym Bias</th>
<th>Approx MSE</th>
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Table 4.3

$n = 10$, $T = 50$  
$\alpha$ is non-normal and $u$ is normal  
$\sigma_\alpha = 1$, $\sigma_u = 0.6$
### Table 4.4

\( n = 10, \ T = 5 \)  \( \alpha \) is normal and \( u \) is non-normal  \( \sigma_\alpha=2, \ \sigma_u=1 \)

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<th>( \theta_2 )</th>
<th>( \gamma_{1u} )</th>
<th>( \gamma_{2u} )</th>
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Table 4.5

$n = 50$, $T = 5$  
$\alpha$ is normal and $u$ is non-normal  
$\sigma_\alpha = 2$, $\sigma_u = 1$
### Table 4.6

\( n = 10, \ T = 50 \) \quad \alpha \text{ is normal and } u \text{ is non-normal} \quad \sigma_\alpha = 2, \ \sigma_u = 1

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Table 4.7

\( n = 10, T = 5 \) Both \( a \) and \( u \) are normal \( \sigma_u = 0.6 \)

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<th>Approx MSE</th>
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### Table 4.8

n = 10, T = 5  \( \alpha \) is normal and u is non-normal  \( \sigma_\alpha=0.5, \sigma_u=2, \lambda=0.76 \)

<table>
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$n=10$, $T=5$  
$\alpha$ is non-normal and $u$ is normal  
$\sigma_\alpha=1.5$, $\sigma_u=2$, $\lambda=0.26$
<table>
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<th>$\gamma_{2u}$</th>
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<th>Asym Bias</th>
<th>Approx MSE</th>
<th>Asym MSE</th>
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$n = 10$, $T = 5$  
$\alpha$ is normal and $u$ is non-normal  
$\sigma_\alpha = 2$, $\sigma_u = 10$, $\lambda = 0.83$
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**Table 4.11**

$n = 10, \ T = 5$  \quad \alpha$ is normal and $u$ is non-normal  \quad $\sigma_\alpha = 2, \ \sigma_u = 20, \ \lambda = 0.95$
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Table 4.12

$n = 10, T = 5$  
Both $\alpha$ and $u$ are non-normal  
$\sigma_\alpha = 5, \sigma_u = 10$
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<th>$\gamma_{2u}$</th>
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<th>Asym Bias</th>
<th>Approx MSE</th>
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Table 4.13

$n = 10, T = 5$ Both \(a\) and \(u\) are non-normal \(\sigma_a=10, \sigma_u=5\)
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Table 4.14
$n = 10, T = 5$, Both $\alpha$ and $u$ are non-normal, $\sigma_\alpha = 10, \sigma_u = 10$